Name: ____________________________________ Date: __________________

COMPLETING THE SQUARE
COMMON CORE ALGEBRA I

The turning point of a parabola and its general shape are relatively easy to determine if the quadratic function is written in its **shifted or vertex form**. Review this in the first exercise.

***Exercise* #1:** Given the function $y=(x-3)^2+2$ do the following.

(a) Give the coordinates of the turning point.

(b) Determine the range by drawing a rough sketch.

The question then is how we take a quadratic of the form $y=ax^2+bx+c$ and put it into its shifted form. This procedure is known as **Completing the Square**. But, it needs some additional review.

***Exercise* #2:** Write each of the following as an equivalent trinomial.

(a) $(x+5)^2$

(b) $(x-1)^2$

(c) $(x+4)^2$

***Exercise* #3:** Given each trinomial in Exercise #2 of the form x^2+bx+c, what is true about the relationship between the value of b and the value of c? Illustrate.

***Exercise* #4:** Each of the following trinomials is a perfect square. Write it in factored (or perfect square) form.

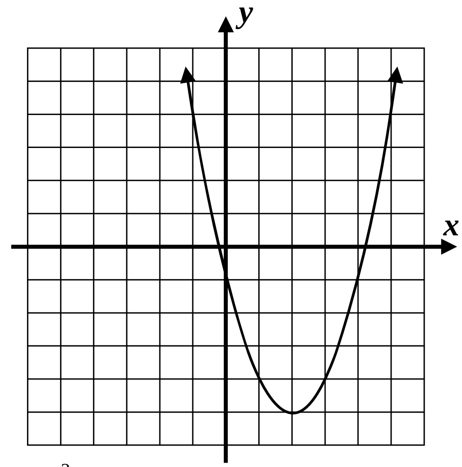
(a) $x^2+20x+100$

(b) x^2-6x+9

(c) x^2+2x+1

We are finally ready to learn the method of **Completing the Square**. This method has many uses, but the one we will work with today is to manipulate equations of quadratics from their **standard form** to their **vertex form**.

***Exercise* #5:** The quadratic $y = x^2 - 4x - 1$ is shown graphed below.



(a) Consider only the binomial $x^2 - 4x$. What would you need to add on to it to create a perfect square trinomial? (See Exercise #3).

(b) In order to add zero to the binomial $x^2 - 4x$, what should we subtract to offset adding 4 to make it a perfect square?

(c) Use the Method of Competing the Square now to rewrite the trinomial $x^2 - 4x - 1$ in an equivalent, shifted form. According to this form, what are the coordinates of the vertex? Verify by examining the graph.

This procedure is what is known as an **algorithm**. In other words, we follow a recipe. Here it is:

COMPLETING THE SQUARE

For the quadratic $y = x^2 + bx + c$ (note that $a = 1$).

1. Find half of the value of *b*, i.e. $\dfrac{b}{2}$ 2. Square it, i.e. $\left(\dfrac{b}{2}\right)^2$ 3. Add and subtract it

There is nothing like practice on these.

***Exercise* #6:** Write each quadratic in vertex form by Completing the Square. Then, identify the quadratic's turning point. The last two problems will involve fractions. Stick with it!

(a) $y = x^2 + 6x - 2$

(b) $y = x^2 - 2x + 11$

(c) $y = x^2 - 10x + 27$

(d) $y = x^2 + 8x$

(e) $y = x^2 + 5x + 4$

(f) $y = x^2 - 9x - 2$

Name: ____________________ Date: ____________

COMPLETING THE SQUARE
COMMON CORE ALGEBRA I HOMEWORK

FLUENCY

1. Find each of the following products in standard form.

(a) $(x+4)^2$ (b) $(x-1)^2$ (c) $(x+8)^2$

(d) $(x-7)^2$ (e) $(x+2)^2$ (f) $(x-10)^2$

2. Each of the following trinomials is a perfect square. Write it in factored form, i.e. $(x+a)^2$ or $(x-a)^2$.

(a) x^2+6x+9 (b) $x^2-22x+121$ (c) $x^2+10x+25$

(d) $x^2+30x+225$ (e) x^2-2x+1 (f) $x^2-18x+81$

3. Place each of the following quadratic functions, written in standard form, into vertex form by completing the square. Then, identify the coordinates of its turning point.

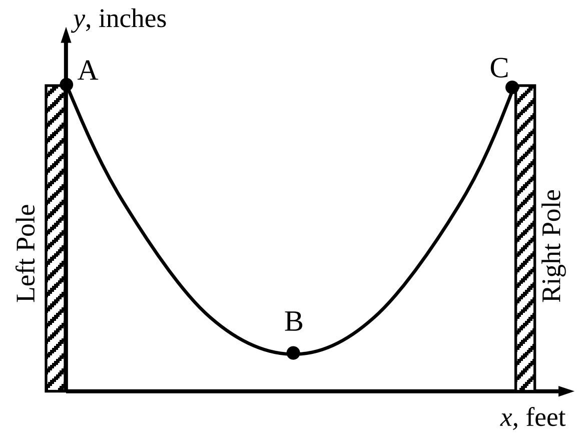
(a) $y=x^2-12x+40$ (b) $y=x^2+4x+14$ (c) $y=x^2-24x+146$

APPLICATIONS

4. A cable is attached at the same height from two poles and hangs between them such that its height above the ground, y, in inches, can be modeled using the equation:

$$y = x^2 - 16x + 67$$

where x represents the horizontal distance from the left pole, in feet.



(a) What height is point A above the ground? Show your work and use proper units.

(b) Write the equation in vertex form.

(c) What is the difference in the heights of points A and B? Show your analysis and include units.

(d) What is the horizontal distance that separates points A and C? Explain your reasoning.

REASONING

5. Use the vertex form of each of the following quadratic functions to determine which has the lowest y-value.

$$y = x^2 - 8x + 6 \qquad\qquad y = x^2 + 6x + 1$$

6. Two quadratic functions are shown below, $f(x)$ and $g(x)$. Determine which has the lower minimum value. Explain how you arrived at your answer.

$$f(x) = x^2 + 10x$$

x	3	4	5	6	7	8	9
$g(x)$	–9	–14	–17	–18	–17	–14	–9