Name: ____________________________________ Date: __________________

POWER FUNCTIONS
COMMON CORE ALGEBRA II

Before we start to analyze polynomials of degree higher than two (quadratics), we first will look at very simple functions known as **power functions**. The formal definition of a power function is given below:

> **POWER FUNCTIONS**
>
> Any function of the form: $f(x) = ax^b$ where a and b are real numbers not equal to zero.

***Exercise* #1:** For each of the following power functions, state the value of a and b by writing the equation in the form $y = ax^b$.

(a) $y = \frac{3}{x^2}$ (b) $y = \frac{1}{7x^3}$ (c) $y = 8\sqrt{x}$ (d) $y = \frac{6}{\sqrt[3]{x}}$

The characteristics of power functions depend on both the value of a and the value of b. The most important, though, is the exponent (the a is simply a vertical stretch of the power function).

***Exercise* #2:** Consider the general power function $y = ax^b$.

(a) What can be said about the y-intercept of any power function if $b > 0$? Illustrate.

(b) What can be said about the y-intercept of any power function if $b < 0$? Illustrate.

For now we will just concentrate on power function where the exponent is a positive whole number.

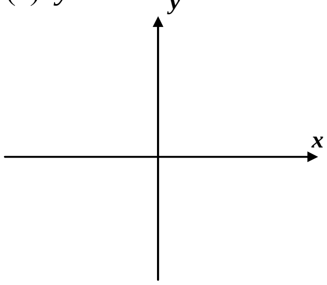
***Exercise* #3:** Using your table, fill in the following values for common power functions.

x	-3	-2	-1	0	1	2	3
x^2							
x^3							
x^4							
x^5							

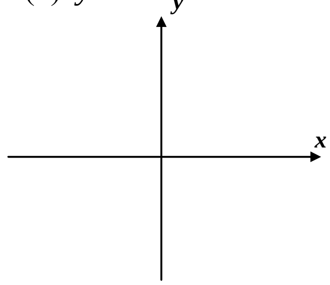
From the previous exercise, we should note that when the power function has an even exponent, then positive and negative INPUTS have the same value. When the power function has an odd exponent, then positive and negative inputs have opposite outputs. Recall this is the definition of **even and odd functions**.

***Exercise* #4:** Using your calculators, sketch the power functions below using the standard viewing window.

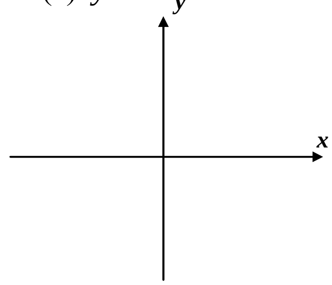
(a) $y = x^2$



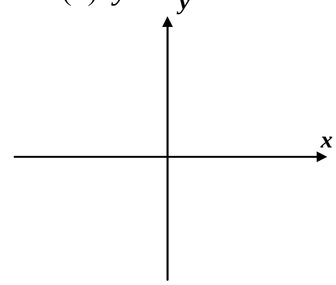
(b) $y = x^3$



(c) $y = x^4$



(d) $y = x^5$



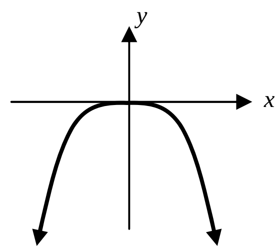
***Exercise* #5:** Which of the following power functions is shown in the graph below? Explain your choice. Do without the use of your calculator.

(1) $y = -4x^7$

(2) $y = -3x^{10}$

(3) $y = 6x^8$

(4) $y = 5x^9$



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<u>The End Behavior of Polynomials</u> – The behavior of polynomials as the input variable gets very large, both positive and negative, is important to understand. We will explore this in the next exercise.

***Exercise* #6:** Consider the two functions $y_1 = x^3 - 2x^2 - 29x + 30$ and $y_2 = x^3$.

(a) Graph these functions using $x_{\min} = -10,\ x_{\max} = 10,\ y_{\min} = -100,\ y_{\max} = 100$

(b) Graph these functions using $x_{\min} = -20,\ x_{\max} = 20,\ y_{\min} = -1000,\ y_{\max} = 1000$

(c) Graph these functions using $x_{\min} = -50,\ x_{\max} = 50,\ y_{\min} = -10000,\ y_{\max} = 10000$

(d) Graph these functions using $x_{\min} = -100,\ x_{\max} = 100,\ y_{\min} = -100000,\ y_{\max} = 100000$

(e) What do you observe about the nature of the two graphs as the viewing window gets larger?

(f) Why is this occurring?

The end behavior (also known as long-run) of any polynomial is dictated by its highest powered term!!!

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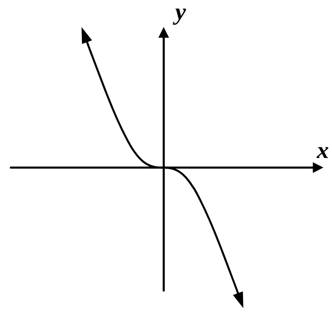
POWER FUNCTIONS
COMMON CORE ALGEBRA II HOMEWORK

FLUENCY

1. **Without** using your calculator, determine which of the following equations could represent the graph shown below. Explain your choice.

(1) $y = x^2$ (2) $y = x^3$

(3) $y = -x^4$ (4) $y = -x^5$



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2. Identify which of the following are power functions. For each that is a power function, write it in the form $y = ax^n$, where a and n are real numbers. Placing them in these forms may take some mindful algebraic manipulation.

(a) $y = 3\sqrt[5]{x}$ (b) $y = 4x^5 - 7$ (c) $y = \dfrac{10}{x^5}$ (d) $y = \dfrac{6x^7}{2x^3}$

(e) $y = x^2 + 2x - 7$ (f) $y = \sqrt{48x^7}$ (g) $y = \sqrt{\dfrac{25}{x^4}}$ (h) $y = 2(x-3)^2$

3. If the point (-3, 8) lies on the graph of a power function with an even exponent, which of the following points must also lie its graph?

(1) $(3, -8)$ (3) $(-3, -8)$

(2) $(3, 8)$ (4) $(8, -3)$

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4. If the point $(-5, 12)$ lies on the graph of a power function with an odd exponent, which of the following points must also lie on its graph?

(1) $(5, -12)$ (2) $(12, -5)$

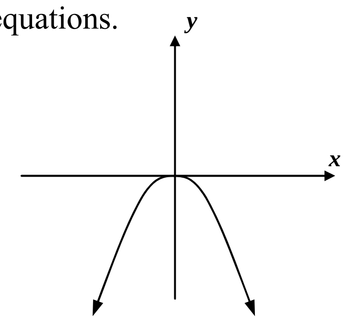
(3) $(-5, -12)$ (4) $(-12, 5)$

5. For each of the following polynomials, give a power function that best represents the end behavior of the polynomial.

(a) $y = 3x^3 - 2x + 12$ (b) $y = 10 - 8x^2$ (c) $y = 6x^5 - 4x^3 + x - 120$

(d) $y = -3x^5 + 2x^4 - 4x + 7$ (e) $y = 5x^4 + 2x^2$ (f) $y = -4x^5 + 8x^7 - 2x^3 + 3$

6 The graph below could be the long-run behavior for which of the following functions? Do this problem **without** graphing each of the following equations.



(1) $y = 2x^2 - 7x + 1$

(2) $y = 4x^3 + 2x^2 - 6x + 4$

(3) $y = -5x^4 + 3x^3 - 2x^2 + x + 9$

(4) $y = -3x^5 - 4x^2 + 2x + 1$

REASONING

7. Let's examine why end-behavior works a little more closely. Consider the functions $f(x) = x^3$ and $g(x) = x^3 + 2x^2 + 7x + 10$.

(a) Fill out the table below for the values of *x* listed. Round your final column to the nearest *hundredth*.

x	$f(x)$	$g(x)$	$\frac{f(x)}{g(x)}$
5			
10			
50			
100			

(b) What number is the ratio in the fourth column approaching as *x* gets larger? What does this tell you about the part of $g(x)$ that can be attributed to the cubic term?