Name: _____ Date: _____

ARITHMETIC AND GEOMETRIC SEQUENCES
ALGEBRA 2 WITH TRIGONOMETRY

There are many important types of sequences in mathematics, but the two most prominent are **arithmetic** and **geometric**. We will begin by defining the arithmetic sequence using a **recursive definition**.

ARITHMETIC SEQUENCE RECURSIVE DEFINITION

Given a_1, then $a_n = a_{n-1} + d$

where *d* is called the **common difference** and can be positive or negative.

***Exercise* #1:** Generate the next three terms of the given arithmetic sequences.

(a) $a_n = a_{n-1} + 6$ with $a_1 = 2$ (b) $a_1 = 8$ and $d = -4$ (c) $t_{n+1} = t_n + \frac{1}{2}$ and $t_1 = \frac{3}{2}$

***Exercise* #2:** Given that $a_1 = -3 + 5i$ and $a_n = a_{n-1} + (6 - i)$, which of the following represents a_4?

(1) $15 + 2i$ (3) $9 + 3i$

(2) $21 + i$ (4) $3 + 4i$

_____

It is important to be able to determine a general term of an arithmetic sequence based on the value of the index variable (the subscript). The next exercise walks you through the thinking process involved.

***Exercise* #3:** Consider $a_n = a_{n-1} + 3$ with $a_1 = 5$.

(a) Determine the value of a_2, a_3, and a_4.

(b) How many times was 3 added to 5 in order to produce a_4?

(c) Use your result from part (b) to quickly find the value of a_{50}.

(d) Write a formula for the n^{th} term of an arithmetic sequence, a_n, based on the first term, a_1, *d* and *n*.

***Exercise* #4:** Given that $a_1 = 6$ and $a_4 = 18$ are members of an arithmetic sequence, determine the value of a_2 and a_{20}.

Geometric sequences are defined very similarly to arithmetic, but with a multiplicative constant instead of an additive one.

GEOMETRIC SEQUENCE RECURSIVE DEFINITION

Given a_1, then $a_n = a_{n-1} \cdot r$

where *r* is called the **common ratio** and can be positive or negative and is often fractional.

***Exercise* #5:** Generate the next three terms of the geometric sequences given below.

(a) $a_1 = 4$ and $r = 2$

(b) $a_n = a_{n-1} \cdot 1/3$ with $a_1 = 9$

(c) $t_n = t_{n-1} \cdot \sqrt{2}$ with $t_1 = 3\sqrt{2}$

And, like arithmetic, we also need to be able to determine any given term of an geometric sequence based on the first value, the common ratio, and the index.

***Exercise* #6:** Consider $a_1 = 2$ and $a_n = a_{n-1} \cdot 3$.

(a) Generate the value of a_4.

(b) How many times did you need to multiply 2 by 3 in order to find a_4.

(c) Determine the value of a_{10}.

(d) Write a formula for the n^{th} term of a geometric sequence, a_n, based on the first term, a_1, r and n.

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ARITHMETIC AND GEOMETRIC SEQUENCES
ALGEBRA 2 WITH TRIGONOMETRY - HOMEWORK

SKILLS

1. Generate the next **three** terms of each arithmetic sequence shown below.

   (a) $a_1 = -2$ and $d = 4$ (b) $a_n = a_{n-1} - 8$ with $a_1 = 10$ (c) $a_1 = 4 + 2i$ and $a_n = a_{n-1} - 2 + 3i$

2. In an arithmetic sequence $t_n = t_{n-1} + 7$. If $t_1 = -5$ determine the values of t_4 and t_{20}. Show the calculations that lead to your answers.

3. If $a_1 = 12$ and $a_n = a_{n-1} - 4$ then which of the following represents the value of a_{40}?

   (1) -148 (3) -144

   (2) -140 (4) -172 _____

4. If $c_1 = -5 + 8i$ and $d = 2 - 3i$ defines an arithmetic sequence of complex numbers then $c_{11} = ?$

   (1) $15 - 22i$ (3) $22 - 33i$

   (2) $17 - 25i$ (4) $-30 + 50i$ _____

5. In an arithmetic sequence of numbers $a_1 = -4$ and $a_6 = 46$. Which of the following is the value of a_{12}?

   (1) 120 (3) 92

   (2) 146 (4) 106 _____

6. The first term of an arithmetic sequence whose common difference is 7 and whose 22^{nd} term is given by $a_{22} = 143$ is which of the following?

   (1) -25 (3) 7

   (2) -4 (4) 28 _____

7. Generate the next **three** terms of each geometric sequence defined below.

(a) $a_1 = -8$ with $r = -1$ (b) $a_n = a_{n-1} \cdot 3/2$ and $a_1 = 16$ (c) $t_{n+1} = t_n \cdot -2$ and $t_1 = 5$

8. Given that $a_1 = 5$ and $a_2 = 15$ are the first two terms of a geometric sequence, determine the values of a_3 and a_{10}. Show the calculations that lead to your answers.

9. If the complex geometric sequence is defined by $c_1 = -7 + 4i$ and $r = i$, where $i = \sqrt{-1}$, then which of the following is the value of c_5?

(1) $7 - 4i$ (3) $-7 + 4i$

(2) $4 - 7i$ (4) $-4 + 7i$ _____

10. In a geometric sequence, it is known that $a_1 = -1$ and $a_4 = 64$. The value of a_{10} is

(1) $-65,536$ (3) 512

(2) 262,144 (4) -4096 _____

APPLICATIONS

11. What would result in more money on the 31st day of the month: (1) Getting paid \$100 on the first day and an extra \$100 per day thereafter or (2) Getting paid 1 penny on the first day and having that amount double each day thereafter? Show calculations for both schemes to justify your answer.