Name: ___________________________________ Date: __________________

FACTORING BASED ON CONJUGATES
COMMON CORE ALGEBRA I

There are a number of different types of factoring techniques. But, each one of them boils down to reversing a product. We begin the lesson today by looking at products of **conjugate binomials**, or binomials of the form $a+b$ and $a-b$.

***Exercise* #1:** Find each of the following products of conjugate pairs. See if you can work out a pattern.

(a) $(x+5)(x-5)$ (b) $(x-2)(x+2)$ (c) $(4x+1)(4x-1)$

(d) $(x+y)(x-y)$ (e) $(2x+3)(2x-3)$ (f) $(5x+2y)(5x-2y)$

What we should see is that if we multiply conjugates, opposites always cancel and instead of getting our expected **trinomial**, we still get a binomial. Specifically.

MULTIPLYING CONJUGATE PAIRS

$$(a+b)(a-b)=a^2-b^2$$

***Exercise* #2:** Use the pattern from Exercise #1 to quickly rewrite the following products.

(a) $(x+6)(x-6)$ (b) $(5x+2)(5x-2)$ (c) $(2x+7y)(2x-7y)$

(d) $(4+x)(4-x)$ (e) $(6+5y)(6-5y)$ (f) $(10x-4y)(10x+4y)$

We now should be able to reverse this multiplication in order to rewrite expressions that are the **difference of perfect squares** into products.

***Exercise* #3:** Write each of the following first in the form $a^2 - b^2$ and then as equivalent products of conjugate pairs.

(a) $x^2 - 81$

(b) $9x^2 - 4$

(c) $25 - y^2$

(d) $4x^2 - 81y^2$

(e) $121x^2 - 1$

(f) $1 - 4x^2$

Never forget that when we factor, we are always rewriting an expression in a form that might look different, but it is ultimately still equivalent to the original.

***Exercise* #4:** Let's take a look at the binomial $x^2 - 9$.

(a) Amelia believes that $x^2 - 9$ can be factored as $(x+1)(x-9)$ while her friend Isabel believes that it is factored as $(x-3)(x+3)$. Fill out the table below to develop evidence as to who is correct. Use technology on your calculator to help.

x	$x^2 - 9$	$(x+1)(x-9)$	$(x-3)(x+3)$
0			
1			
2			
3			

(b) By multiplying out their respective factors, show which of the two friends has the correct factorization. Use the Distributive Property Twice.

Amelia: $(x+1)(x-9)$

Isabel: $(x-3)(x+3)$

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FACTORING BASED ON CONJUGATE PAIRS
COMMON CORE ALGEBRA I HOMEWORK

FLUENCY

1. Use the fact that the product of conjugates follows the pattern $(a+b)(a-b)=a^2-b^2$ to quickly find the following products in standard form.

(a) $(x-5)(x+5)$ (b) $(x+7)(x-7)$ (c) $(2-x)(2+x)$

(d) $(3x+2)(3x-2)$ (e) $(4x+1)(4x-1)$ (f) $(2x+1)(2x-1)$

(g) $(5-4x)(5+4x)$ (h) $(x^2-2)(x^2+2)$ (i) $(x^3+4)(x^3-4)$

2. Write each of the following binomials as an equivalent product of conjugates.

(a) x^2-16 (b) x^2-100 (c) x^2-1

(d) x^2-25 (e) $4-x^2$ (f) $9-x^2$

(g) $4x^2-1$ (h) $16x^2-49$ (i) $1-25x^2$

(j) x^2-9y^2 (k) $81-4t^2$ (l) x^4-36

APPLICATIONS

3. A square is changed into a new rectangle by increasing its width by 2 inches and decreasing its length by 2 inches. Make sure to draw pictures to help you solve these problems!

(a) If the original square had a side length of 8 inches, find its area and the area of the new rectangle. How many square inches larger is the square's area?

(b) If the original square had a side length of 20 inches, find its area and the area of the new rectangle. How many square inches larger is the square's area?

(c) If the square had a side length of x inches, show that its area will always be four square inches more than the area of the new rectangle.

REASONING

4. Consider the numerical expression $51^2 - 49^2$.

(a) Use your calculator to find the numerical value of this expression.

(b) Can you use facts about conjugate pairs to show why this difference should work out to be the answer from (a)?

5. Consider the following expression $(x+2)(x-2)-(x+4)(x-4)$.

(a) Using your calculator, determine the value of this expression for various values of x.

x	$(x+2)(x-2)-(x+4)(x-4)$
-2	
-1	
0	
1	
2	

(b) Algebraically show that this product has a constant value (seen in (a)) regardless of the value of x.