Name: ___________________________________ Date: _________________

GEOMETRIC SERIES
ALGEBRA 2 WITH TRIGONOMETRY

Just as we can sum the terms of an arithmetic sequence to generate an arithmetic series, we can also sum the terms of a geometric sequence to generate a **geometric series**.

***Exercise* #1:** Given a geometric series defined by the recursive formula $a_1 = 3$ and $a_n = a_{n-1} \cdot 2$, which of the following is the value of $S_5 = \sum_{i=1}^{5} a_i$?

(1) 106 (3) 93

(2) 75 (4) 35 __________

The sum of a finite number of geometric sequence terms is less obvious than that for an arithmetic series, but can be found nonetheless. The next exercise derives the formula for finding this sum.

***Exercise* #2:** Recall that for a geometric sequence, the *n*th term is given by $a_n = a_1 \cdot r^{n-1}$. Thus, the general form of an geometric series is given below.

$$S_n = a_1 + a_1 r + a_1 r^2 + \cdots + a_1 r^{n-2} + a_1 r^{n-1}$$

(a) Write an expression below for the product of r and S_n.

$r \cdot S_n =$

(b) Find, in simplest form, the value of $S_n - r \cdot S_n$ in terms of a_1, r, and n.

$S_n - r \cdot S_n =$

(c) Write both sides of the equation in (b) in their factored form.

(d) From the equation in part (c), find a formula for S_n in terms of a_1, r, and n.

***Exercise* #3:** Which of the following represents the sum of a geometric series with 8 terms whose first term is 3 and whose common ratio is 4?

(1) 32,756 (3) 42,560

(2) 28,765 (4) 65,535 __________

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SUM OF A FINITE GEOMETRIC SERIES

For a geometric series defined by its first term, a_1, and its common ratio, *r*, the sum of *n* terms is given by:

$$S_n = \frac{a_1\left(1-r^n\right)}{1-r}$$

***Exercise* #4:** Find the value of the geometric series shown below. Show the calculations that lead to your final answer.

$$6+12+24+\cdots+768$$

***Exercise* #5:** Maria places \$500 at the beginning of each year into an account that earns 5% interest compounded annually. Maria would like to determine how much money is in her account after she has made her \$500 deposit at the beginning of the 11th year.

(a) Determine a formula for the amount, $A(t)$, that a given \$500 has grown to *t*-years after it was placed into this account.

(b) Using this formula, determine the amount that \$500 deposited in years 1, 2, 10, and 11 have earned at the beginning of the 11th year.

(c) Based on (b), write a geometric sum representing the amount of money in Maria's account after the 11th year deposit.

(d) Evaluate the sum in (c) using the formula above.

***Exercise* #6:** A person places 1 penny in a piggy bank on the first day of the month, 2 pennies on the second day, 4 pennies on the third, and so on. Will this person be a millionaire at the end of a 31 day month? Show the calculations that lead to your answer.

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Geometric Series
Algebra 2 with Trigonometry - Homework

Skills

1. Find the sums of geometric series with the following properties:

(a) $a_1 = 6$, $r = 3$ and $n = 8$ (b) $a_1 = 20$, $r = \frac{1}{2}$, and $n = 6$ (c) $a_1 = -5$, $r = -2$, and $n = 10$

2. If the geometric series $54 + 36 + \cdots + \frac{128}{27}$ has seven terms in its sum then the value of the sum is

(1) $\frac{4118}{27}$ (3) $\frac{1370}{9}$

(2) $\frac{1274}{3}$ (4) $\frac{8241}{54}$ __________

3. A geometric series has a first term of 32 and a final term of $-\frac{1}{4}$ and a common ratio of $-\frac{1}{2}$. The value of this series is

(1) 19.75 (3) 22.5

(2) 16.25 (4) 21.25 __________

4. Which of the following represents the value of $\sum_{i=0}^{8} 256\left(\frac{3}{2}\right)^i$?

(1) 19,171 (3) 22,341

(2) 12,610 (4) 8,956 __________

5. A geometric series whose first term is 3 and whose common ratio is 4 sums to 4095. The number of terms in this sum is

(1) 8 (3) 6

(2) 5 (4) 4 __________

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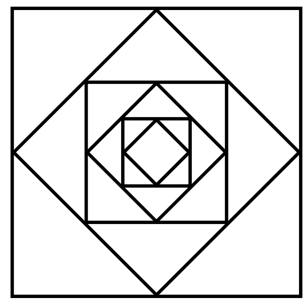
6. Find the sum of the geometric series shown below. Show the work that leads to your answer.

$$27+9+3+\cdots\frac{1}{729}$$

APPLICATIONS

7. In the picture shown at the right, the outer most square has an area of 16 square inches. All other squares are constructed by connecting the midpoints of the sides of the square it is inscribed within. Find the sum of the areas of all of the squares shown.



8. A college savings account is constructed so that \$1000 is placed the account on January 1st of each year with a guaranteed 3% yearly return in interest, applied at the end of each year to the balance in the account. If this is repeatedly done, how much money is in the account after the \$1000 is deposited at the beginning of the 19th year? Show the sum that leads to your answer as well as relevant calculations.

9. A ball is dropped from 16 feet above a hard surface. After each time it hits the surface, it rebounds to a height that is $3/4$ of its previous maximum height. What is the total vertical distance, to the nearest foot, the ball has traveled when it strikes the ground for the 10th time? Write out the first five terms of this sum to help visualize.

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