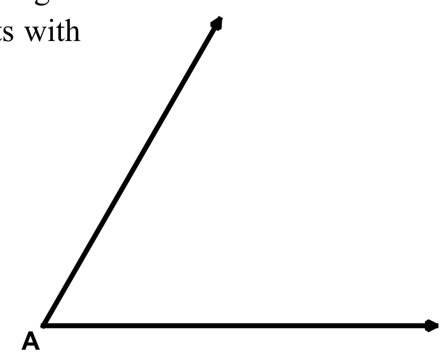
Name: ________________________________ Date: ________________

BISECTING AN ANGLE
COMMON CORE GEOMETRY

One of the fundamental assumptions (or axioms) of geometry is that every angle has a unique bisector, i.e. a single line that divides the angle into two angles with equal measures. Constructing the angle bisector is relatively easy but is an extremely important construction.

***Exercise* #1:** Given $\angle A$ shown, do the following:

(a) Draw an arc with a center of *A* that intersects the two rays of the angle. Mark the intersection points *B* and *C*. What is true about both points with respect to point *A*?



(b) Using *B* and *C* as centers, draw arcs of the same radius so that they intersect. Label the intersection as point *D*. Note that the arcs here do not need to be the same radius as that drawn in (a). What's true about point *D*?

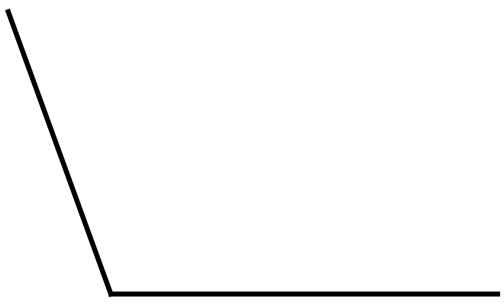
(c) Draw a ray starting at *A* and passing through *D*. This ray now bisects $\angle A$. Use tracing paper to verify that the two angles, i.e. $\angle BAD$ and $\angle CAD$ are congruent.

(d) Fill in the reasons for the formal proof below showing why $\overline{AD}$ must bisect $\angle BAC$.

Statements	Reasons
(1) $\overline{AB} \cong \overline{AC}$	(1) ____________________
(2) $\overline{BD} \cong \overline{CD}$	(2) ____________________
(3) $\overline{AD} \cong \overline{AD}$	(3) ____________________
(4) $\Delta ABD \cong \Delta ACD$	(4) ____________________
(5) $\angle BAD \cong \angle CAD$	(5) ____________________
(6) $\overline{AD}$ bisects $\angle BAC$	(6) ____________________ ____________________

***Exercise* #2:** Construct the angle bisector for each angle shown below. Leave all construction marks and verify that the angle has been bisected by using tracing paper.

(a)



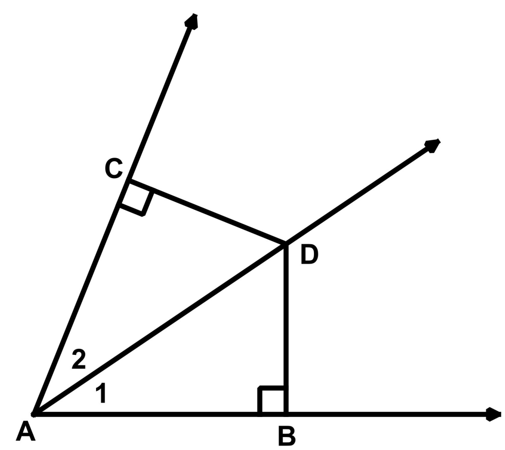
(b)

In the last unit, we proved that points that lie along the angle bisector are equidistant from the sides of the angle. The key to understanding this is another fundamental assumption in geometry:

> The **shortest distance** from a point to a line is the length of the perpendicular line segment that can be drawn from the point to the line.

We will quickly review the thinking behind the proof that points along the angle bisector are equidistant from the sides of the angle.

***Exercise* #3:** In the diagram below, $\angle CAB$ has been bisected by $\overrightarrow{AD}$, where D is just some arbitrary point on the angle bisector such that $\overline{CD} \perp \overline{AC}$ and $\overline{BD} \perp \overline{AB}$.



(a) What triangle congruence theorem could be used to show that ΔACD is congruent to ΔABD? Explain your thinking.

(b) What is be true about $\overline{CD}$ and $\overline{BD}$? Why must this be true? How does this show that D is equidistant from $\overrightarrow{AC}$ and $\overrightarrow{AB}$?

(c) Construct a line that represents all points equidistant from the sides of $\angle BAD$. Leave all construction marks.

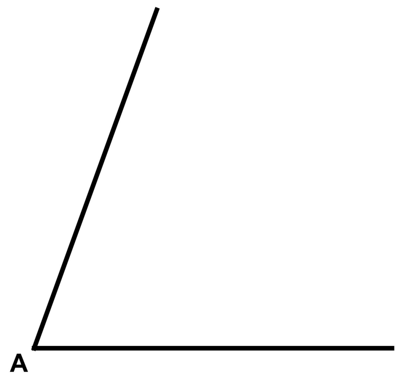
Name: ___________________________________ Date: _________________

BISECTING AN ANGLE
COMMON CORE GEOMETRY HOMEWORK

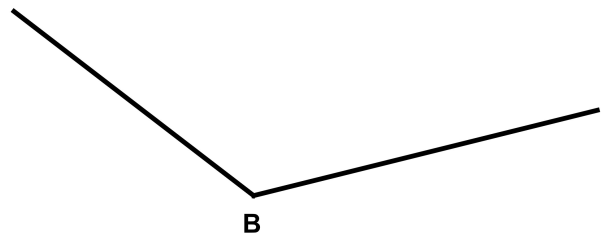
MEASUREMENT AND CONSTRUCTION

1. Construct the angle bisector of each angle shown below. Leave all construction marks.

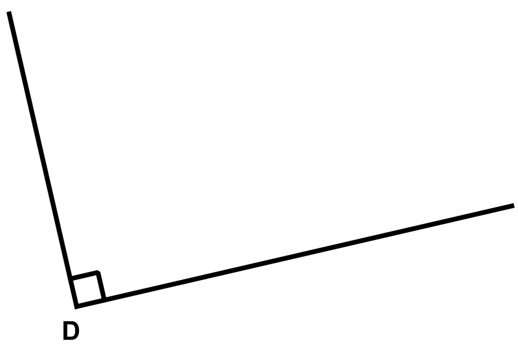
(a)



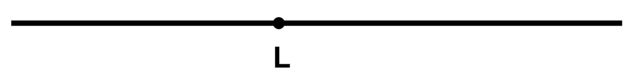
(b)



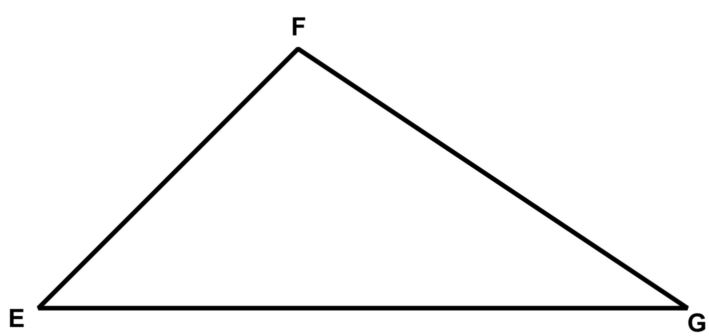
(c)



(d)



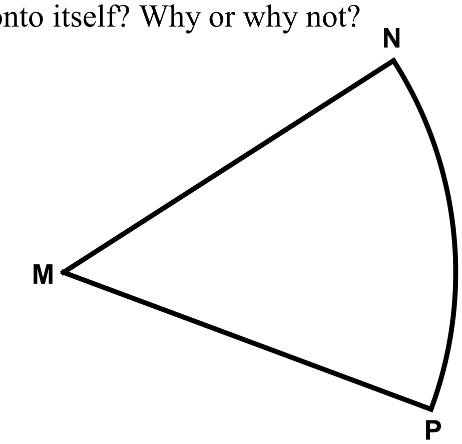
2. For ΔEFG shown below, construct the angle bisectors of all three angles. If you do this carefully, all three angle bisectors will intersect at a common point (will be **concurrent**).



3. Rectangle $ABCD$ is shown below. Construct the angle bisector of $\angle D$. Leave all construction marks.

4. A **diagonal** of a rectangle (or any polygon) is a line segment that connects any two non-adjacent vertices of the rectangle. In the rectangle in #3, do the diagonals bisects the right angles of this rectangle? Justify.

5. A sector of a circle is shown below. Construct a line that represents all points equidistant from $\overline{MN}$ and $\overline{MP}$. Would a reflection across this line map the figure onto itself? Why or why not?



6. **Construction Review:** Construct a line that passes through the point shown below that is perpendicular to the given line segment. Label the intersection of the two lines B. Leave all construction marks.

The length of $\overline{AB}$ represents what important distance?