Name: ____________________________________ Date: _________________

SYMMETRIES OF A FIGURE
COMMON CORE GEOMETRY

We've now seen how the idea of **congruence** can be formally defined using **rigid motions**. We will end our discussion, for now, about rigid motions by looking at very specialized ones known as **symmetries**.

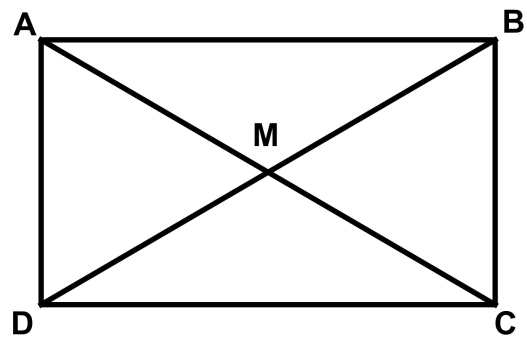
SYMMETRIES

A **transformation** that **maps a figure onto itself** (also known as carrying a figure onto itself) is known as a **symmetry of the figure**.

***Exercise* #1:** Explain why a symmetry transformation must be a rigid motion?

We will investigate symmetries of figures in this lesson primarily with using reflections and rotations, but not translations.

***Exercise* #2:** Given rectangle *ABCD* shown below, which is not also a square, answer the following questions. The diagonals of the rectangle, $\overline{AC}$ and $\overline{BD}$, have been drawn and intersect at point *M*.

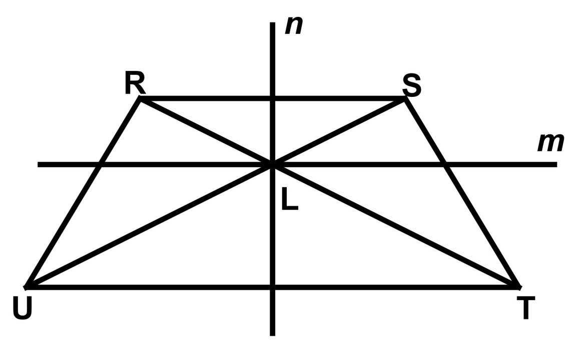


(a) What is the minimum angle you could rotate *ABCD* about *M* to have it mapped on itself? Explain. Use tracing paper if needed.

(b) Are there any lines of symmetry? How would you describe them? Draw them as best as you can on the diagram.

***Exercise* #3:** Trapezoid *RSTU* is drawn below with $\overline{RU} \cong \overline{ST}$. Line *n* is the perpendicular bisector of $\overline{RS}$ and $\overline{UT}$, and line *m* is perpendicular to *n* through L. Which of the following transformations would carry *RSTU* onto itself? Verify using tracing paper.

(1) a rotation about *L* by 180°
(2) a rotation about *L* by 90°
(3) a reflection across line *n*
(4) a reflection across line *m*



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We can analyze the symmetries of any figure, but some of the most interesting are the **regular polygons**. First, their definition:

REGULAR POLYGONS

A **regular polygon** is any polygon whose sides have all equal lengths and whose angles all have equal measures.

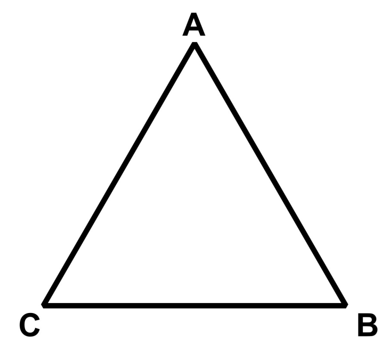
***Exercise* #4:** Give the standard name for each of the following:

(a) a regular triangle

(b) a regular quadrilateral

Regular polygons tend to have many symmetries. Let's first look at an equilateral triangle.

***Exercise* #5:** Consider the equilateral (regular) triangle shown below.



(a) Describe the lines of symmetry of this figure? How many will it have? Draw them.

(b) What kind of rotational symmetry does this triangle have? Discuss both the center of rotation and the angles of rotation.

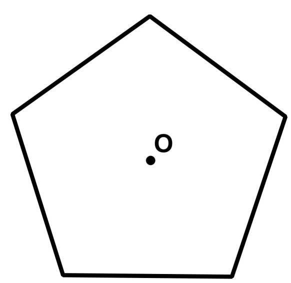
***Exercise* #6:** Given the regular pentagon shown below, which of the following angles of rotation about center point O would *not* carry the figure onto itself?

(1) 72°

(2) 144°

(3) 224°

(4) 288°



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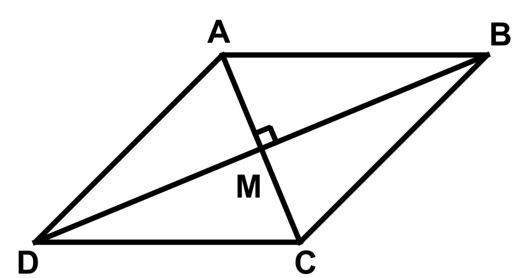
***Exercise* #7:** The non-rectangular, parallelogram $ABCD$ with four equal sides is shown below (known as a rhombus). The diagonals of $ABCD$ are perpendicular and meet at M. Explain why a counterclockwise rotation of 90° about M will *not* map the figure onto itself.



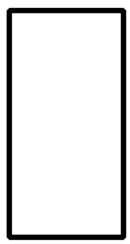
Name: ___________________________________ Date: _________________

Symmetries of a Figure
Common Core Geometry Homework

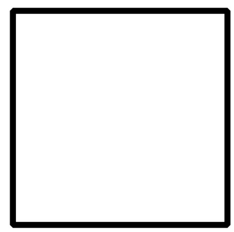
Measurement and Construction

1. For each of the following figures, draw all lines of reflection that could be used to map the figure onto itself. Verify using tracing paper. There is one figure that has no lines of symmetry.

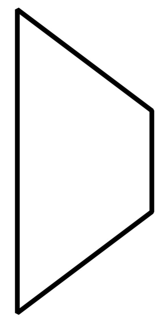
(a)



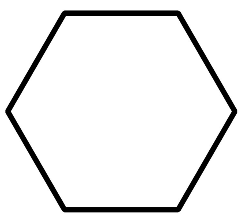
(b)



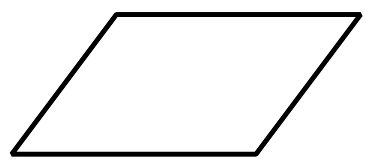
(c)



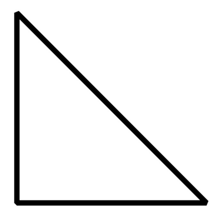
(d)



(e)



(f)

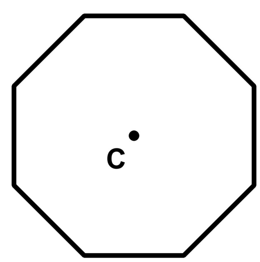


Problem Solving

2. In the regular octagon shown below, which of the following is the minimum angle of rotation about center point *C* that will carry the figure onto itself?

(1) 15° (3) 45°

(2) 30° (4) 90°



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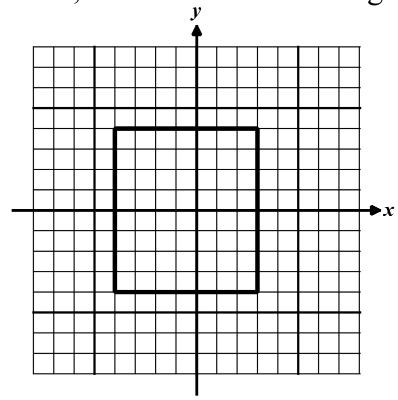
3. In the rectangle shown on the coordinate grid below, which of the following transformations would map the figure onto itself?

(1) a reflection across the *x*-axis

(2) a reflection across the *y*-axis

(3) a 180° rotation about the origin

(4) a 90° rotation about the origin



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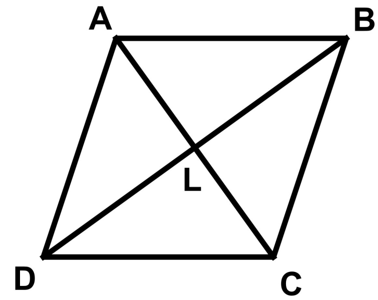
4. Given the non-square rhombus shown below, which of the following is true about its symmetries?



(1) It has reflective symmetry across $\overleftrightarrow{AC}$ and rotational symmetry of 90° about L.

(2) It has reflective symmetry across $\overleftrightarrow{BD}$ and rotational symmetry of 90° about L.

(3) It has reflective symmetry across $\overleftrightarrow{AC}$ and $\overleftrightarrow{BD}$ and no rotational symmetry.

(4) It has reflective symmetry across $\overleftrightarrow{AC}$ and $\overleftrightarrow{BD}$ and rotational symmetry of 180° about L.

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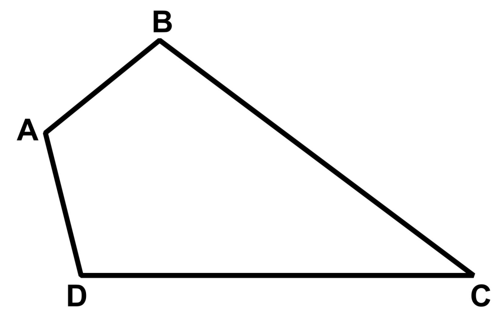
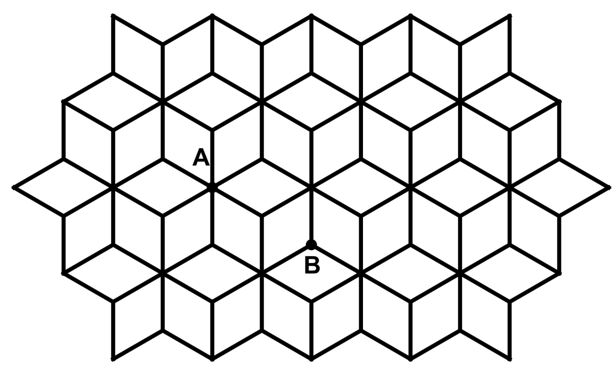
5. A **kite** is defined as a quadrilateral with two pairs of equal, **adjacent** (touching) sides. In the diagram below, a kite is shown with $AB = AD$ and $BC = DC$. Over what line(s) could kite ABCD be reflected onto itself? Drawn the line(s). Verify with tracing paper if necessary.



6. A "tiling" of the plane occurs when we fill the **entire, infinite** plane with a repeated pattern. The image to the right **shows a portion** of one such tiling. The pattern extends forever in all directions.



(a) Give all angles of rotational symmetry between 0 and 360° about the two marked points.

Point A: Point B:

(b) Draw and label line m such that m is neither horizontal nor vertical and is a line of reflective symmetry of the tiling.

Reasoning

7. Explain why any circle would have an infinite number of lines of reflection that would carry the circle onto itself.