

PARTITIONING A LINE SEGMENT
COMMON CORE GEOMETRY



In the last lesson, we saw the **Side Splitter Theorem** which said that **parallel lines** that intersect sides of a triangle will **partition (divide up)** those sides **proportionally**. We can take advantage of this thinking when **partitioning** a line segment in the **coordinate plane**.

Exercise #1: Segment \overline{AB} lies in the coordinate plane with point P on segment \overline{AB} such that $AP:PB = 1:1$. If the endpoints are at $A(-2, 13)$ and $B(8, 5)$ then answer the following questions.

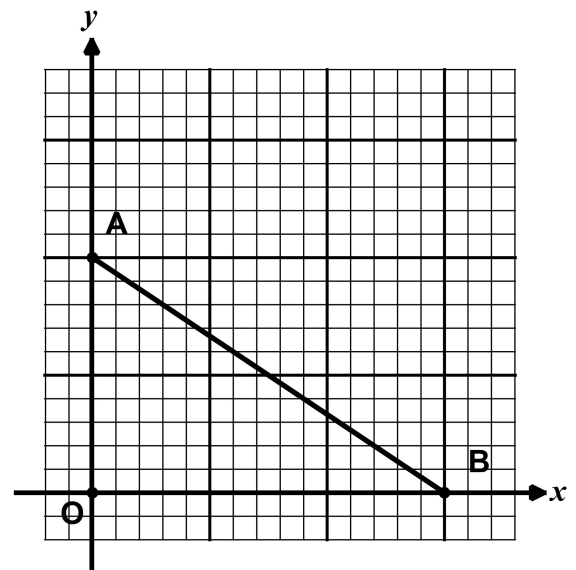
- (a) What can you say based on the givens about the lengths of \overline{AP} and \overline{PB} ? Draw a picture to help.
- (b) What special point is P on line segment \overline{AB} ? Find the coordinates of P .

When the ratio of the partitioning is not quite this simple, then we need to rely on other tools, like the Side Splitter Theorem. We will see how to employ that in the next exercise.

Exercise #2: In the following graph, segment \overline{AB} has endpoints at $A(0, 10)$ and $B(15, 0)$. We want to locate point P on \overline{AB} such that $AP:PB = 3:2$. The origin has been marked as point O .

- (a) Draw $\triangle AOB$ below without the grid and mark the approximate location of P on \overline{AB} .

- (b) On your diagram above, draw a vertical line from P down to side \overline{OB} . Mark the intersection C . Why must \overline{PC} be parallel to \overline{OA} ?



- (c) What is the ratio of $OC:CB$? Explain. Then, algebraically find the coordinates of C .
- (d) What are the coordinates of P ?



For obvious reasons, it is easier to partition horizontal or vertical line segments because their lengths are fairly easy to work with. Every line segment can be partitioned this way. Let's look at another example.

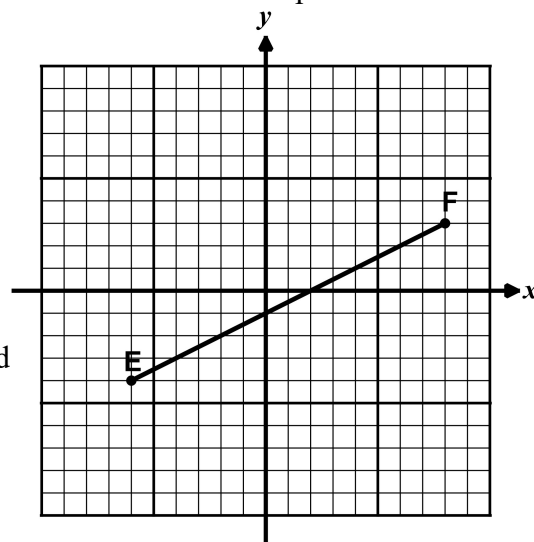
Exercise #3: On the grid to the right \overline{EF} is plotted with endpoints at:

$$E(-6, -4) \text{ and } F(8, 3)$$

We want to find the coordinates of point G lying on \overline{EF} such that:

$$EG : GF = 3 : 4$$

(a) Draw a right triangle below with \overline{EF} as the hypotenuse and horizontal and vertical legs. Label the lengths of the legs.



(b) Find the coordinates of G . Show how you arrived at your answer.

These partitioning problems are more difficult, but certainly not impossible, with larger measurements and no grid paper to work with. Let's see how it's done.

Exercise #4: Segment \overline{CD} has point E located on it such that $CE : ED = 3 : 5$. If the endpoints are located at $C(-5, -6)$ and $D(11, 18)$ then find the coordinates of E . Show how you arrived at your answer.

Exercise #5: Segment \overline{EF} has endpoints at $E(-7, 14)$ and $F(11, 5)$. Point P lies on \overline{EF} at $P(-3, 12)$. Which of the following is the ratio of EP to PF ?

(1) 2:7

(3) 4:9

(2) 9:4

(4) 7:2



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PROBLEM SOLVING

1. If point P lies on segment \overline{AB} with endpoints at $A(-5, 11)$ and $B(1, 5)$ such that $AP : PB = 1 : 1$ then P must have coordinates of

(1) $(-4, 16)$ (3) $(3, 3)$

(2) $(-2, 8)$ (4) $(6, 4)$ _____

2. Segment \overline{RS} has endpoints at $R(5, -3)$ and $S(5, 13)$. Point Q lies on \overline{RS} such that $RQ : QS = 3 : 1$. Which of the following must be the coordinates of Q ?

(1) $(5, 1)$ (3) $(5, 9)$

(2) $(17, 3)$ (4) $(6, 4)$ _____

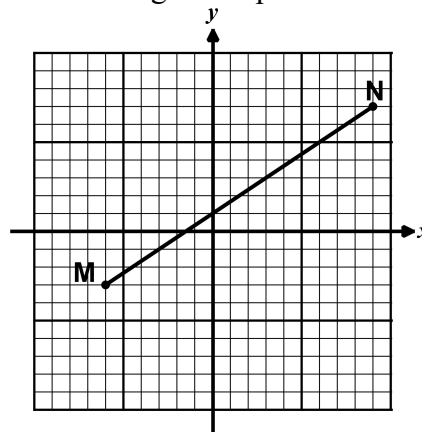
3. Segment \overline{MN} has endpoints at $M(-6, -3)$ and $N(9, 7)$. Point Q lies on \overline{MN} such that $MQ : QN = 3 : 2$. Which of the following are the coordinates of Q ? Use of the grid is optional.

(1) $(3, 3)$

(3) $(-3, -1)$

(2) $(6, 5)$

(4) $(0, 1)$



4. Segment \overline{AB} has endpoints at $A(-14, 12)$ and $B(10, -4)$. Point C lies on \overline{AB} at $C(-5, 6)$. Which of the following represents the ratio of AC to BC ?

(1) $2 : 5$ (3) $3 : 7$

(2) $4 : 9$ (4) $3 : 5$ _____



5. Segment \overline{AB} has endpoints at $A(-4, -8)$ and $B(10, 13)$. If point P lies on \overline{AB} such that $AP:BP = 2:5$, then find the coordinates of P . Show all your work.

6. Segment \overline{GH} has endpoints at $G(-2, 9)$ and $H(10, 3)$. Point J lies on \overline{GH} between G and H such that $GJ:GH = 1:3$. Find the coordinates of J . Watch out, there is something slightly different here.

7. Segment \overline{MN} has endpoints at $M(-12, -5)$ and $N(8, 10)$. A third point Q is located on \overline{MN} such that $MQ:QN = 2:3$. Answer the following questions.

(a) Find the coordinates of Q . Show how you arrived at your answer.

(b) Verify that $MQ:QN = 2:3$ by finding the lengths of \overline{MQ} and \overline{QN} using the distance formula.

Length of \overline{MQ} :

Length of \overline{QN} :

Ratio:

(c) Write the equation of the line, in **point-slope form**, that passes through Q and is perpendicular to \overline{MN} . Show the work involved in finding your answer.

