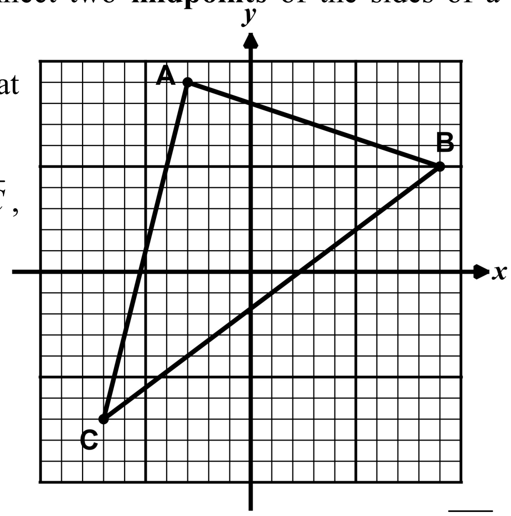
Name: ___________________________________ Date: _________________

THE MIDPOINTS OF A TRIANGLE
COMMON CORE GEOMETRY

Now that we've learned a bit about parallelograms, we are going to take a slight diversion back to the work of triangles to investigate an interesting thing that happens when you connect two **midpoints** of the sides of a triangle with a line segment.

***Exercise* #1:** In the diagram shown, ΔABC is drawn with vertices at $A(-3,9)$, $B(9,5)$, and $C(-7,-7)$.



(a) Using the midpoint formula, find the midpoints of sides $\overline{AB}$ and $\overline{AC}$, then plot and label them *D* and *E* respectively.

Midpoint of $\overline{AB}$:
(point *D*)

Midpoint of $\overline{AC}$:
(point *E*)

(b) Draw in segment $\overline{DE}$. Then, using the distance and slope formulas, find the length and slope of both $\overline{DE}$ and $\overline{BC}$. What observations can you make about these two segments?

Length of $\overline{DE}$:

Length of $\overline{BC}$:

Observations:

Slope of $\overline{DE}$:

Slope of $\overline{BC}$:

This is a remarkable fact. **When you connect the midpoints of any two sides** of a triangle, the segment created is **parallel to** and **half the length of the third side**.

***Exercise* #2:** If a triangle with side lengths of 10, 14, and 18 has the midpoints of all three sides connected by segments, what must be the perimeter of the triangle formed? Draw a picture to help explain your work.

***Exercise* #3:** ΔLMN has vertices at $L(6,11)$, $M(13,7)$ and $N(2,1)$. If the midpoints of sides $\overline{LM}$ and $\overline{NM}$ are connected with a segment, which of the following would be its slope?

(1) $\frac{5}{2}$ (3) $-\frac{4}{7}$

(2) $\frac{6}{11}$ (4) $-\frac{3}{2}$

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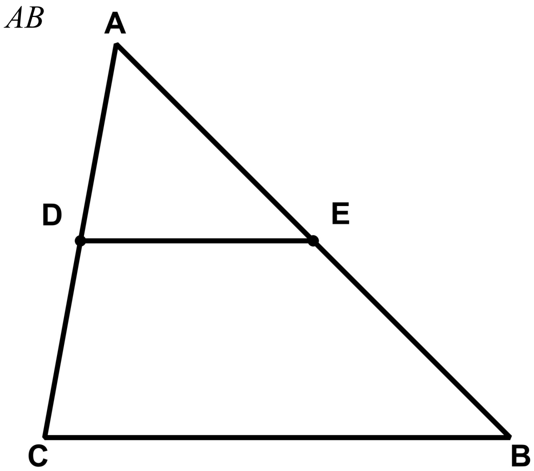
These two facts about the segment joining the midpoints of two sides of a triangle can be proven using Euclidean logic and what we know about parallelograms. The reasoning behind the proof will be presented in the next exercise.

***Exercise* #4:** In the diagram shown, ΔABC has the midpoints of sides $\overline{AB}$ and $\overline{AC}$ marked with points E and D as shown.



(a) Using only a compass and straightedge, extend $\overline{DE}$ through E to point F such that $\overline{DE} \cong \overline{EF}$. Leave all construction marks.

(b) Draw in segment $\overline{BF}$.

(c) Give an argument for why ΔAED must be congruent to ΔBEF.

(d) Explain why, since $\Delta AED \cong \Delta BEF$, $\overline{BF}$ must be parallel to $\overline{AC}$. Why does this then lead to the fact that $\overline{BF}$ must be parallel to $\overline{CD}$?

(e) Why must $\overline{BF}$ and $\overline{CD}$ be the same length? Give a careful explanation based on what you know, not just what the picture looks like.

(f) Why can you now conclude that $DFBC$ is a parallelogram? How does this conclusion then result in the two properties we've observed about the segment connecting the midpoints (in this case $\overline{DE}$)?

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THE MIDPOINTS OF A TRIANGLE
COMMON CORE GEOMETRY HOMEWORK

PROBLEM SOLVING

1. If the midpoints of two sides of a triangle are connected with a segment then

(1) the segment is half the length of the third side and perpendicular to it
(2) the segment is twice the length of the third side and parallel to it
(3) the segment is half the length of the third side and parallel to it
(4) the segment is twice the length of the third side and perpendicular to it

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2. A right triangle has legs with lengths of 10 and 24 inches. If the midpoints of all three sides of this triangle were connected, the resulting right triangle would have a hypotenuse with a length of

(1) 5 inches (3) 12 inches

(2) 8 inches (4) 13 inches

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3. If a triangle has a perimeter of 36 inches, which of the following would be the perimeter of the triangle formed by connecting its midpoints?

(1) 18 inches (3) 24 inches

(2) 20 inches (4) 72 inches

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4. In ΔRST, the vertices have coordinates of $R(-1,10)$, $S(5,4)$ and $T(-4,-2)$. If the midpoints of sides $\overline{RS}$ and $\overline{RT}$ were connected with a segment, what would the slope of this segment be?

(1) -1 (3) $\frac{2}{3}$

(2) $-\frac{5}{2}$ (4) 4

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5. For ΔDEF, the vertices have coordinates of $D(4,14), E(6,1)$ and $F(-4,6)$. Which of the following would represent the equation of the line that passes through the midpoints of sides $\overline{DE}$ and $\overline{FD}$?

(1) $y=-\frac{1}{2}x+10$ (3) $y=-\frac{1}{2}x+5$

(2) $y=-\frac{3}{2}x-4$ (4) $y=-\frac{3}{2}x+8$

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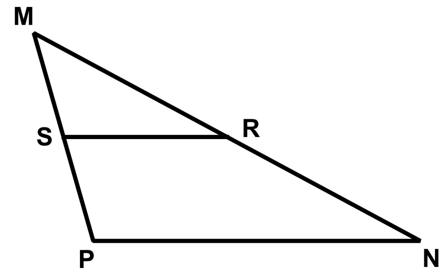
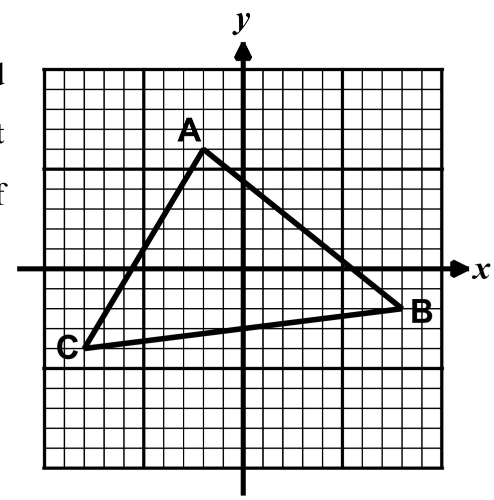
REASONING

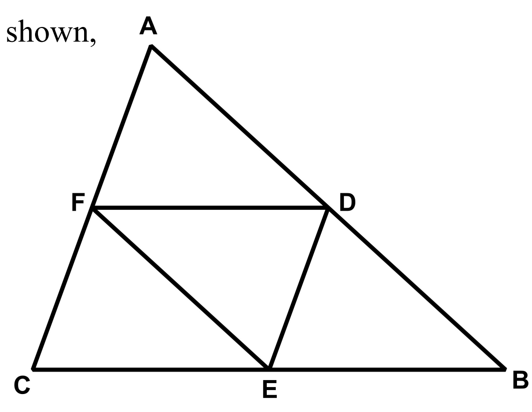
6. In the diagram of ΔMNP shown, the midpoints of sides $\overline{MN}$ and $\overline{MP}$ have been connected to form segment $\overline{SR}$. Explain why quadrilateral $SRNP$ must be a trapezoid.



7. For ΔABC shown, with vertices at $A(-2,6)$, $B(8,-2)$ and $C(-8,-4)$, show using coordinate geometry that the segment connecting the midpoints of sides $\overline{AC}$ and $\overline{BC}$ is half the length of side $\overline{AB}$. Show all work below.



8. Given ΔABC with midpoints of sides D, E, F connected as shown, explain why $\Delta AFD \cong \Delta FCE \cong \Delta DEB \cong \Delta EDF$.



9. Given what we observed in #8, how does the area of ΔAFD above compare to the area of ΔABC? Explain your answer.