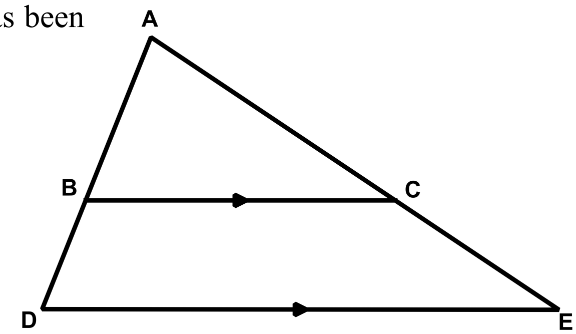
Name: ___________________________________ Date: _________________

THE SIDE SPLITTER THEOREM
COMMON CORE GEOMETRY

In the next few lessons we will be looking at specialty applications of similarity. One of the most famous and useful is known as the **Side Splitter Theorem**. The first exercise will help investigate this theorem.

***Exercise* #1:** In the diagram shown of ΔADE, line segment $\overline{BC}$ has been drawn such that it is parallel to $\overline{DE}$.



(a) Justify why ΔABC is similar to ΔADE.

(b) If $AB = 6$, $AD = 10$, and $AC = 9$, set up and solve a proportion to find the value of AE. Also, find the length of segment $\overline{CE}$.

(c) Find the following ratios of lengths in their simplest forms and compare. What do you notice?

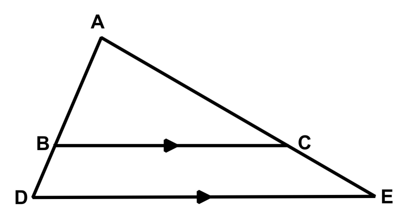
$\frac{AB}{BD} =$ $\qquad\qquad$ $\frac{AC}{CE} =$

So, we see that the parallel line segment drawn **divides up each side proportionally**.

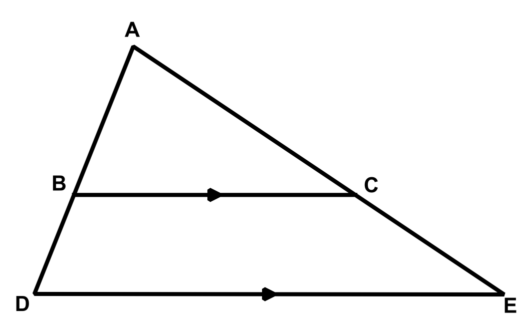
THE SIDE SPLITTER THEOREM

A line drawn parallel to any side of a triangle that intersects the other two sides will divide them proportionally, i.e.

$$\frac{AB}{BD} = \frac{AC}{CE}$$



***Exercise* #2:** Given the diagram, *algebraically* prove the Side Splitter Theorem.



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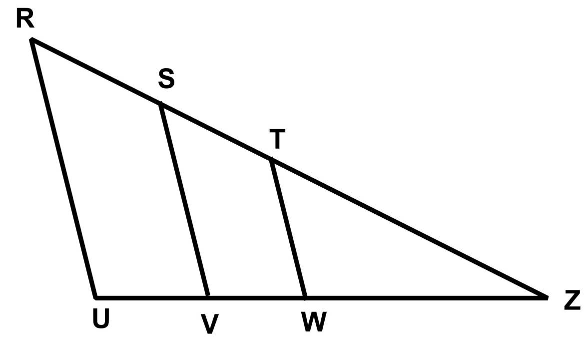
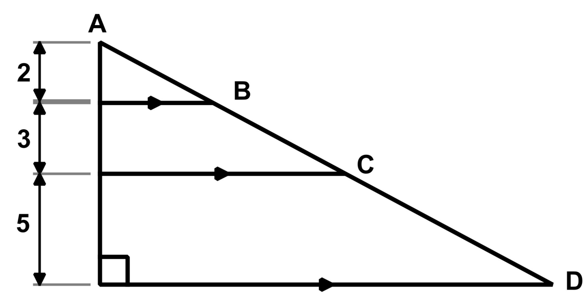
The Side Splitter Theorem makes solving for portions of the sides of a triangle split by parallel lines relatively easy, as the following exercises will illustrate.

***Exercise* #3:** In the following diagram, $\overline{RU} \parallel \overline{SV} \parallel \overline{TW}$. If $RS = 15, VW = 6, TZ = 25,$ and $WZ = 15$ find the lengths of segments $\overline{ST}$ and $\overline{UV}$. Show the work that leads to your answers.



***Exercise* #4:** A right triangle has had one of its legs divided into segments of lengths 2, 3, and 5 as shown. Lines parallel to the other leg are then drawn to the hypotenuse $\overline{AD}$, dividing it into three segments. If $AD = 30$, then determine the lengths of AB, BC and CD. Show the work that leads to your answer.

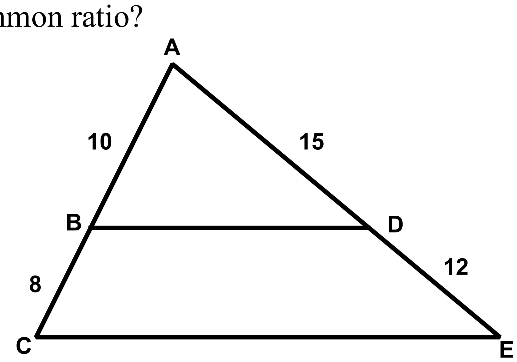


It should be noted that the **converse** of the **Side Splitter Theorem** is also true. In other words, if the sides are cut proportionally, then the lines must be parallel. Let's explore the reasoning just a bit in the next exercise.

***Exercise* #5:** In the following diagram it is known that $AB = 10$, $BC = 8$, $AD = 15$ and $DE = 12$.

(a) Verify that $\frac{AB}{BC} = \frac{AD}{DE}$

(b) Is $\frac{AC}{AB} = \frac{AE}{AD}$? If so, what is the common ratio?



(c) Based on (b) give a dilation that would show that $\Delta ABD \sim \Delta ACE$.

(d) How does knowing $\Delta ABD \sim \Delta ACE$ show that the lines are parallel?

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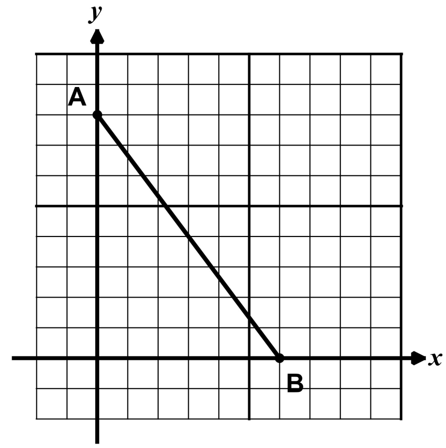
THE SIDE SPLITTER THEOREM
COMMON CORE GEOMETRY HOMEWORK

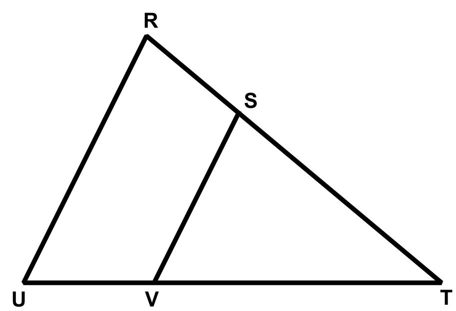
MEASUREMENT AND CONSTRUCTION

1. Line segment $\overline{AB}$ is drawn on the grid with endpoints at $A(0,8)$ and $B(6,0)$. Using only a straightedge and the grid itself, divide $\overline{AB}$ into four congruent segments. Explain how you did this and why it works.



PROBLEM SOLVING

2. In ΔRUT, S and V have been located on its sides such that $\overline{SV} \parallel \overline{RU}$. If $ST = 12$, $RT = 20$ and $VT = 15$ then which of the following is the length of $\overline{UV}$?



(1) 9
(2) 10
(3) 25
(4) 35

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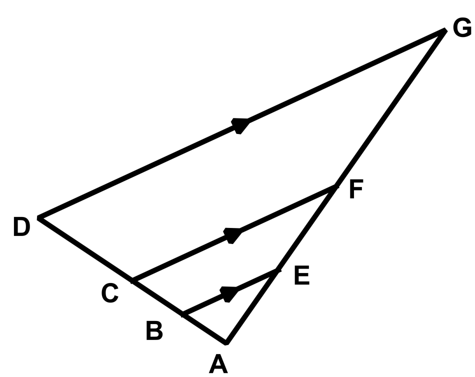
3. In the following diagram, not drawn to scale, $\overline{BC}$ is twice the length of $\overline{AB}$ and $\overline{CD}$ is twice the length of $\overline{BC}$. If $\overline{AG}$ is 140 inches long, how many inches long is $\overline{EF}$? Show how you arrived at your answer.



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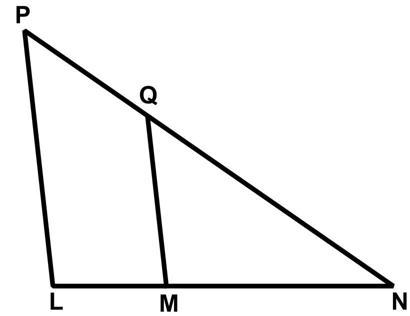
4. Which of the following segment lengths would justify the claim that $\overline{PL} \parallel \overline{QM}$?



(1) $LM = 8, MN = 12, PQ = 10,$ and $QN = 14$

(2) $LM = 5, MN = 10, PQ = 8,$ and $QN = 18$

(3) $LM = 6, MN = 10, PQ = 9,$ and $QN = 15$

(4) $LM = 10, MN = 15, PQ = 12,$ and $QN = 20$

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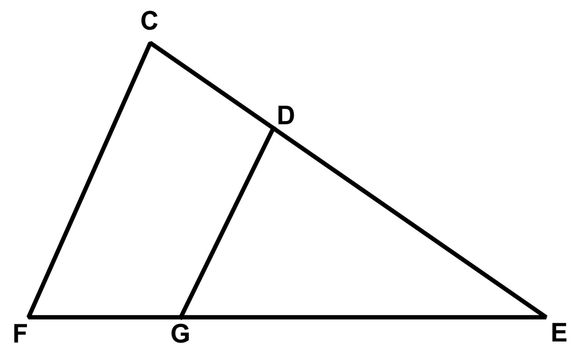
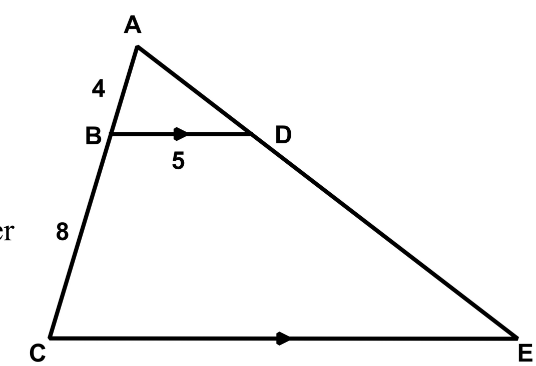
5. In the following diagram, $\overline{CF} \parallel \overline{DG}$. Segment $\overline{DE}$ has a length that is 4 inches greater than the length of $\overline{CD}$. If $FG = 15$ inches and $FE = 36$ inches, then determine the length of $\overline{CD}$.



Reasoning

6. Francine was working with a problem involving triangles and parallel lines. She was given the following diagram with $AB = 4, BC = 8,$ and $BD = 5$. She was asked to solve for the length of $\overline{CE}$.



(a) Francine *incorrectly* solves for CE as follows:

$$\frac{4}{8} = \frac{5}{CE} \Rightarrow 4CE = 40 \Rightarrow CE = 10$$

Explain why Francine is incorrect in using the Side Splitter Theorem here.

(b) Show the correct solution for the length of $\overline{CE}$ below.

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