Name: _____ Date: _____

THE DISCRIMINANT OF A QUADRATIC
COMMON CORE ALGEBRA II

Since the roots of a quadratic can be found using $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ the quantity under the square root, $b^2 - 4ac$, truly dictates what type of numbers the roots of a quadratic (and its *x*-intercepts or zeroes) turn out to be. It reduces down to four cases which will be explored in *Exercise* #1.

***Exercise* #1:** For each of the following quadratics, calculate its discriminant, its roots, and state the number and nature (whether they are rational, irrational or imaginary) of the roots.

(a) Case I – The Discriminant is a Perfect Square: $x^2 + 3x - 10 = 0$.

$D = b^2 - 4ac =$ Roots: Number and Nature:

(b) Case II – The Discriminant is Not a Perfect Square: $x^2 - 6x + 7 = 0$.

$D = b^2 - 4ac =$ Roots: Number and Nature:

(c) Case III – The Discriminant is Equal to Zero: $x^2 - 10x + 25 = 0$.

$D = b^2 - 4ac =$ Roots: Number and Nature:

(d) Case IV – The Discriminant is Less than Zero: $x^2 - 8x + 20 = 0$

$D = b^2 - 4ac =$ Roots: Number and Nature:

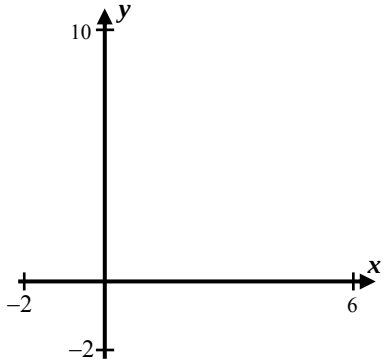
In the last lesson, we explored Case IV extensively. In the case where the discriminant is negative, the roots of the quadratic are **imaginary** and it does not have *x*-intercepts (i.e. it does not cross the *x*-axis).

***Exercise* #2:** By using only the discriminant, determine the number and nature of the roots of each of the following quadratics.

(a) $2x^2 + 7x - 4 = 0$ (b) $x^2 - 8x + 25 = 0$ (c) $4x^2 + 4x + 1 = 0$

(d) $x^2 + 6x + 15 = 0$ (e) $4x^2 - 4x - 7 = 0$ (f) $3x^2 - 7x + 2 = 0$

***Exercise* #3:** Consider the quadratic function whose equation is $y = x^2 - 4x + 4$. Determine the number of *x*-intercepts of this quadratic from the value of its discriminant. Then, sketch its graph on the axes given. We say that this parabola is **tangent** to the *x*-axis.



***Exercise* #4:** Which of the following parabolas has two unequal, rational *x*-intercepts?

(1) $y = x^2 - 2x - 1$ (3) $y = x^2 - 8x + 16$

(2) $y = x^2 + 2x - 15$ (4) $y = x^2 - 3x + 5$ _____

***Exercise* #5:** For what values of *a* will the parabola $y = ax^2 + 4x + 2$ not cross the *x*-axis?

Name: _____ Date: _____

THE DISCRIMINANT OF A QUADRATIC
COMMON CORE ALGEBRA II HOMEWORK

SKILLS

1. For each of the following quadratic equations, determine the number and the nature of the roots by first calculating the quadratic's discriminant.

   (a) $2x^2 + 4x + 5 = 0$ (b) $9x^2 - 12x + 4 = 0$ (c) $4x^2 - 13x + 3 = 0$

   (d) $x^2 + 8x + 11 = 0$ (e) $4x^2 + 4x - 7 = 0$ (f) $36x^2 - 12x + 1 = 0$

   (g) $-3x^2 + 4x - 8 = 0$ (h) $3x^2 + 8x + 4 = 0$ (i) $x^2 + 8x + 41 = 0$

2. The roots of $x^2 + 4x - 7 = 0$ are

   (1) unequal and rational (3) unequal and irrational

   (2) unequal and imaginary (4) equal and rational _____

3. Which of the following quadratics has imaginary roots?

   (1) $x^2 + 3x - 5 = 0$ (3) $2x^2 - 3x + 1 = 0$

   (2) $x^2 + 6x + 10 = 0$ (4) $x^2 - 7x + 10 = 0$ _____

4. Which of the following quadratics, when graphed, would touch the *x*-axis exactly once?

   (1) $y = x^2 - 2x - 3$ (3) $y = x^2 + 5x - 2$

   (2) $y = 2x^2 + 3x + 7$ (4) $y = x^2 - 12x + 36$ _____

5. If graphed, which of the following parabolas would lie entirely below the *x*-axis?

(1) $y = x^2 + 5x + 10$ (3) $y = -2x^2 + 6x - 5$

(2) $y = -2x^2 - 5x + 3$ (4) $y = x^2 + 6x + 9$

_____

6. Which parabola below, when graphed, would cross the *x*-axis at two unequal irrational locations?

(1) $y = 2x^2 + 11x + 12$ (3) $y = 9x^2 - 6x + 1$

(2) $y = x^2 + 2x - 4$ (4) $y = 2x^2 + 4x + 9$

_____

REASONING

7. Determine all values of *a* that will cause the parabola $y = ax^2 + 10x + 1$ to cross the *x*-axis at two distinct locations.

8. Consider the parabola whose equation is $y = x^2 - 4x$ and the line whose equation is $y = 2x + b$, where *b* is some unknown constant. Determine the value of *b* such that the line and the parabola will intersect at exactly one location. Then, sketch the system of equations on the axes below. Label their intersection point.