Name: ____________________________________ Date: __________________

Exponential Modeling with Percent Growth and Decay
Common Core Algebra II

Exponential functions are very important in modeling a variety of real world phenomena because certain things either increase or decrease by **fixed percentages** over given units of time. You looked at this in Common Core Algebra I and in this lesson we will review much of what you saw.

***Exercise* #1:** Suppose that you deposit money into a savings account that receives 5% interest per year on the amount of money that is in the account for that year. Assume that you deposit \$400 into the account initially.

(a) How much will the savings account increase by over the course of the year?

(b) How much money is in the account at the end of the year?

(c) By what single number could you have multiplied the \$400 by in order to calculate your answer in part (b)?

(d) Using your answer from part (c), determine the amount of money in the account after 2 and 10 years. Round all answers to the nearest cent when needed.

(e) Give an equation for the amount in the savings account $S(t)$ as a function of the number of years since the \$400 was invested.

(f) Using a table on your calculator determine, to the nearest year, how long it will take for the initial investment of \$400 to double. Provide evidence to support your answer.

The thinking process from *Exercise* #1 can be generalized to any situation where a quantity is increased by a fixed percentage over a fixed interval of time. This pattern is summarized below:

Increasing Exponential Models

If quantity Q increases by a fixed percentage p, in **decimal form**, over **a unit increase in time**, t, then Q can be modeled by

$$Q(t) = Q_0(1+p)^t$$

where Q_0 represents the amount of Q present at $t=0$.

***Exercise* #2:** Which of the following gives the savings S in an account if \$250 was invested at an interest rate of 3% per year?

(1) $S = 250(4)^t$ (3) $S = (1.03)^t + 250$

(2) $S = 250(1.03)^t$ (4) $S = 250(1.3)^t$

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Decreasing exponentials are developed in the same way, but have the percent subtracted, rather than added, to the base of 100%. Just remember, you are ultimately multiplying by the percent of the original that you will have after the time period elapses.

***Exercise* #3:** State the multiplier (base) you would need to multiply by in order to decrease a quantity by the given percent listed.

(a) 10% (b) 2% (c) 25% (d) 0.5%

DECREASING EXPONENTIAL MODELS

If quantity Q decreases by a fixed percentage p, in **decimal form**, over **a unit increase in time**, t, then Q can be modeled by

$$Q(t) = Q_0(1-p)^t$$

where Q_0 represents the amount of Q present at $t = 0$.

***Exercise* #4:** If the population of a town is decreasing by 4% per year and started with 12,500 residents, which of the following is its projected population in 10 years? Show the exponential model you use to solve this problem.

(1) 9,230 (3) 18,503

(2) 76 (4) 8,310

***Exercise* #5:** The stock price of WindpowerInc is increasing at a rate of 4% per week. Its initial value was $20 per share. On the other hand, the stock price in GerbilEnergy is crashing (losing value) at a rate of 11% per week. If its price was $120 per share when Windpower was at $20, after how many weeks will the stock prices be the same? Model both stock prices using exponential functions. Then, find when the stock prices will be equal graphically. Draw a well labeled graph to justify your solution.

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EXPONENTIAL MODELING WITH PERCENT GROWTH AND DECAY
COMMON CORE ALGEBRA II HOMEWORK

APPLICATIONS

1. If $130 is invested in a savings account that earns 4% interest per year, which of the following is closest to the amount in the account at the end of 10 years?

(1) $218 (3) $168

(2) $192 (4) $324 __________

2. A population of 50 fruit flies is increasing at a rate of 6% per day. Which of the following is closest to the number of days it will take for the fruit fly population to double?

(1) 18 (3) 12

(2) 6 (4) 28 __________

3. If a radioactive substance is quickly decaying at a rate of 13% per hour approximately how much of a 200 pound sample remains after one day?

(1) 7.1 pounds (3) 25.6 pounds

(2) 2.3 pounds (4) 15.6 pounds __________

4. A population of llamas stranded on a desert island is decreasing due to a food shortage by 6% per year. If the population of llamas started out at 350, how many are left on the island 10 years later?

(1) 257 (3) 102

(2) 58 (4) 189 __________

5. Which of the following equations would model a population with an initial size of 625 that is growing at an annual rate of 8.5%?

(1) $P=625(8.5)^t$ (3) $P=1.085^t+625$

(2) $P=625(1.085)^t$ (4) $P=8.5t^2+625$ __________

6. The acceleration of an object falling through the air will decrease at a rate of 15% per second due to air resistance. If the initial acceleration due to gravity is 9.8 meters per second per second, which of the following equations best models the acceleration *t* seconds after the object begins falling?

(1) $a=15-9.8t^2$ (3) $a=9.8(1.15)^t$

(2) $a=\dfrac{9.8}{15t}$ (4) $a=9.8(0.85)^t$ __________

7. Red Hook has a population of 6,200 people and is growing at a rate of 8% per year. Rhinebeck has a population of 8,750 and is growing at a rate of 6% per year. In how many years, to the nearest year, will Red Hook have a greater population than Rhinebeck? Show the equation or inequality you are solving and solve it graphically.

8. A warm glass of water, initially at 120 degrees Fahrenheit, is placed in a refrigerator at 34 degrees Fahrenheit and its temperature is seen to decrease according to the exponential function

$T(h) = 86(0.83)^h + 34$ where h is the number of hours since placed in the refrigerator

(a) Verify that the temperature starts at 120 degrees Fahrenheit by evaluating $T(0)$.

(b) Using your calculator, sketch a graph of T below for all values of h on the interval $0 \le h \le 24$. Be sure to label your y-axis and y-intercept.

(c) After how many hours will the temperature be at 50 degrees Fahrenheit? State your answer to the nearest *hundredth* of an hour. Illustrate your answer on the graph your drew in (b).

REASONING

9. Percents combine in strange ways that don't seem to make sense at first. It would seem that if a population grows by 5% per year for 10 years, then it should grow in total by 50% over a decade. But this isn't true. Start with a population of 100. If it grows at 5% per year for 10 years, what is its population after 10 years? What percent growth does this represent?