Name: ___________________________________ Date: __________________

COMPOUND INTEREST
COMMON CORE ALGEBRA II

In the worlds of investment and debt, interest is added onto a principal in what is known as **compound interest**. The percent rate is typically given on a yearly basis, but could be applied more than once a year. This is known as the **compounding frequency**. Let's take a look at a typical problem to understand how the compounding frequency changes how interest is applied.

***Exercise* #1:** A person invests \$500 in an account that earns a **nominal yearly interest rate** of 4%.

(a) How much would this investment be worth in 10 years if the **compounding frequency** was once per year? Show the calculation you use.

(b) If, on the other hand, the interest was applied four times per year (known as quarterly compounding), why would it not make sense to multiply by 1.04 each quarter?

(c) If you were told that an investment earned 4% per year, how much would you assume was earned per quarter? Why?

(d) Using your answer from part (c), calculate how much the investment would be worth after 10 years of quarterly compounding? Show your calculation.

So, the pattern is fairly straightforward. For a **shorter compounding period**, we get to **apply the interest more often**, but at a **lower rate**.

***Exercise* #2:** How much would \$1000 invested at a nominal 2% yearly rate, compounded monthly, be worth in 20 years? Show the calculations that lead to your answer.

(1) \$1485.95 (3) \$1033.87

(2) \$1491.33 (4) \$1045.32 _________

This pattern is formalized in a classic formula from economics that we will look at in the next exercise.

***Exercise* #3:** For an investment with the following parameters, write a formula for the amount the investment is worth, A, after t-years.

P = amount initially invested

r = nominal yearly rate (percent in decimal form)

n = number of compounds per year

$A(t) =$

The rate in *Exercise* #1 was referred to as **nominal** (**in name only**). It's known as this because you **effectively** earn more than this rate if the compounding period is more than once per year. Because of this, bankers refer to the **effective rate**, or the rate you would receive if compounded just once per year. Let's investigate this.

***Exercise* #4:** An investment with a nominal yearly rate of 5% is compounded at different frequencies. Give the **effective** yearly rate, accurate to two decimal places, for each of the following compounding frequencies. Show your calculation.

(a) Quarterly

(b) Monthly

(c) Daily

We could compound at smaller and smaller frequency intervals, eventually compounding all moments of time. In our formula from *Exercise* #3, we would be letting n approach infinity. Interestingly enough, this gives rise to **continuous compounding** and the use of the natural base **e** in the famous **continuous compound interest formula.**

CONTINUOUS COMPOUND INTEREST

For an initial principal, P, compounded continuously at a nominal yearly rate of r, the investment would be worth an amount A given by:

$$A(t) = Pe^{rt}$$

***Exercise* #5:** A person invests \$350 in a bank account that promises a nominal yearly rate of 2% continuously compounded.

(a) Write an equation for the amount this investment would be worth after t-years.

(b) How much would the investment be worth after 20 years?

(c) Algebraically determine the time it will take for the investment to reach \$400. Round to the nearest tenth of a year.

(d) What is the effective annual rate for this investment? Round to the nearest hundredth of a percent.

Name: ____________________________________ Date: __________________

COMPOUND INTEREST
COMMON CORE ALGEBRA II HOMEWORK

APPLICATIONS

1. The value of an initial investment of \$400 at 3% nominal yearly interest compounded quarterly can be modeled using which of the following equations, where t is the number of years since the investment was made?

(1) $A=400(1.0075)^{4t}$ (3) $A=400(1.03)^{4t}$

(2) $A=400(1.0075)^{t}$ (4) $A=400(1.0303)^{4t}$

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2. Which of the following represents the value of an investment with a principal of \$1500 with a nominal yearly interest rate of 2.5% compounded monthly after 5 years?

(1) \$1,697.11 (3) \$4,178.22

(2) \$1,699.50 (4) \$5,168.71

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3. Franco invests \$4,500 in an account that earns a 3.8% nominal yearly interest rate compounded continuously. If he withdraws the profit from the investment after 5 years, how much has he earned on his investment?

(1) \$858.92 (3) \$922.50

(2) \$912.59 (4) \$941.62

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4. An investment that returns a nominal 4.2% yearly rate, but is compounded quarterly, has an effective yearly rate closest to

(1) 4.21% (3) 4.27%

(2) 4.24% (4) 4.32%

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5. If an investment's value can be modeled with $A=325\left(1+\frac{.027}{12}\right)^{12t}$ then which of the following describes the investment?

(1) The investment has a nominal rate of 27% compounded every 12 years.
(2) The investment has a nominal rate of 2.7% compounded every 12 years.
(3) The investment has a nominal rate of 27% compounded 12 times per year.
(4) The investment has a nominal rate of 2.7% compounded 12 times per year.

____

6. An investment of \$500 is made at 2.8% nominal yearly interest compounded quarterly.

(a) Write an equation that models the amount A the investment is worth t-years after the principal has been invested.

(b) How much is the investment worth after 10 years?

(c) Algebraically determine the number of years it will take for the investment to be reach a worth of \$800. Round to the nearest *hundredth*.

(d) Why does it make more sense to round your answer in (c) to the nearest quarter? State the final answer rounded to the nearest quarter.

REASONING

7. The formula $A = P\left(1+\frac{r}{n}\right)^{nt}$ can be rearranged using properties of exponents as $A = P\left(\left(1+\frac{r}{n}\right)^{n}\right)^{t}$. Explain what the term $\left(1+\frac{r}{n}\right)^{n}$ helps to calculate.

8. The formula $A = Pe^{rt}$ calculates the amount an investment earning a nominal rate of r compounded continuously is worth. Show that the amount of time it takes for the investment to double in value is given by the expression $\frac{\ln 2}{r}$.