Name: ______________________________________ Date: ___________________

THE METHOD OF COMMON BASES
COMMON CORE ALGEBRA II

There are very few algebraic techniques that **do not involve technology** to solve equations that contain **exponential expressions**. In this lesson we will look at one of the few, known as **The Method of Common Bases**.

***Exercise* #1:** Solve each of the following simple exponential equations by writing each side of the equation using **a common base**.

(a) $2^x = 16$ (b) $3^x = 27$ (c) $5^x = \frac{1}{25}$ (d) $16^x = 4$

In each of these cases, even the last, more challenging one, we could manipulate the right-hand side of the equation so that it shared a **common base** with the left-hand side of the equation. We can exploit this fact by manipulating both sides so that they have a common base. First, though, we need to review an exponent law.

***Exercise* #2:** Simplify each of the following exponential expressions.

(a) $\left(2^3\right)^x$ (b) $\left(3^2\right)^{4x}$ (c) $\left(5^{-1}\right)^{3x-7}$ (d) $\left(4^{-3}\right)^{1-x^2}$

***Exercise* #3:** Solve each of the following equations by finding a common base for each side.

(a) $8^x = 32$ (b) $9^{2x+1} = 27$ (c) $125^x = \left(\frac{1}{25}\right)^{4-x}$

***Exercise* #4:** Which of the following represents the solution set to the equation $2^{x^2-3} = 64$?

(1) $\{\pm 3\}$ (3) $\{\pm\sqrt{11}\}$

(2) $\{0, 3\}$ (4) $\{\pm\sqrt{35}\}$

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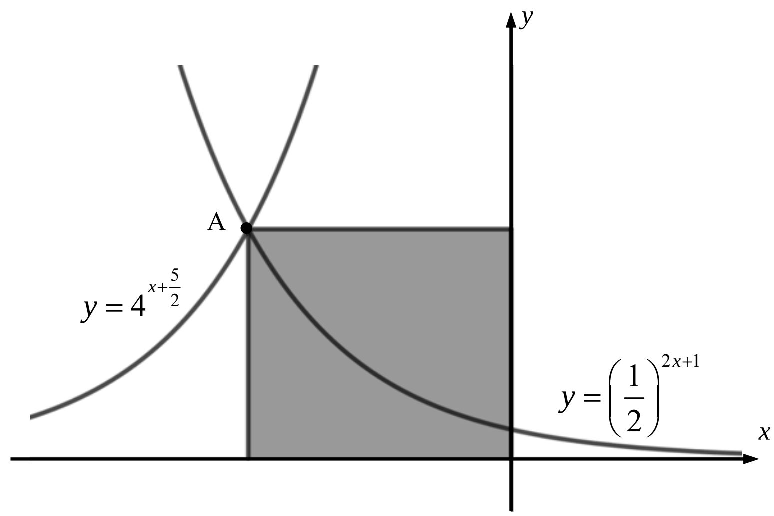
This technique can be used in any situation where all bases involved can be written with a common base. In a practical sense, this is rather rare. Yet, these types of algebraic manipulations help us see the **structure** in **exponential expressions**. Try to tackle the next, more challenging, problem.

Exercise #5: Two exponential curves, $y=4^{x+\frac{5}{2}}$ and $y=\left(\frac{1}{2}\right)^{2x+1}$ are shown below. They intersect at point A. A rectangle has one vertex at the origin and the other at A as shown. We want to find its area.

(a) Fundamentally, what do we need to know about a rectangle to find its area?

(b) How would knowing the coordinates of point A help us find the area?



(c) Find the area of the rectangle algebraically using the Method of Common Bases. Show your work carefully.

Exercise #6: At what *x* coordinate will the graph of $y=25^{x-a}$ intersect the graph of $y=\left(\frac{1}{125}\right)^{3x+1}$? Show the work that leads to your choice.

(1) $x=\frac{5a-1}{3}$

(3) $x=\frac{-2a+1}{5}$

(2) $x=\frac{2a-3}{11}$

(4) $x=\frac{5a+3}{2}$

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THE METHOD OF COMMON BASES
COMMON CORE ALGEBRA II HOMEWORK

FLUENCY

1. Solve each of the following exponential equations using the Method of Common Bases. Each equation will result in a linear equation with one solution. Check your answers.

(a) $3^{2x-5}=9$ (b) $2^{3x+7}=16$ (c) $5^{4x-5}=\frac{1}{125}$

(d) $8^{x}=4^{2x+1}$ (e) $216^{x-2}=\left(\frac{1}{1296}\right)^{3x-2}$ (f) $\left(\frac{1}{25}\right)^{x+15}=3125^{\frac{3}{5}x-1}$

2. *Algebraically* determine the intersection point of the two exponential functions shown below. Recall that most systems of equations are solved by substitution.

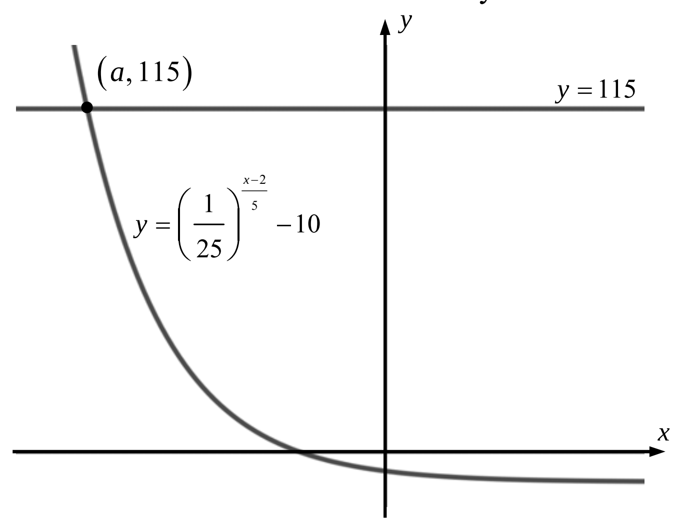
$$y=8^{x-1} \text{ and } y=4^{2x-3}$$

3. *Algebraically* determine the **zeroes** of the exponential function $f\left(x\right)=2^{2x-9}-32$. Recall that the reason it is known as a zero is because the **output is zero**.

APPLICATIONS

4. One hundred must be raised to what power in order to be equal to a million cubed? Solve this problem using the Method of Common Bases. Show the algebra you do to find your solution.

5. The exponential function $y=\left(\frac{1}{25}\right)^{\frac{x-2}{5}}-10$ is shown graphed along with the horizontal line $y=115$. Their intersection point is $(a,115)$. Use the Method of Common Bases to find the value of a. Show your work.



REASONING

6 The Method of Common Bases works because exponential functions are one-to-one, i.e. if the outputs are the same, then the inputs must also be the same. This is what allows us to say that if $2^x=2^3$, then x must be equal to 3. But it doesn't always work out so easily.

If $x^2=5^2$, can we say that x must be 5? Could it be anything else? Why does this not work out as easily as the exponential case?