Name: _____ Date: _____

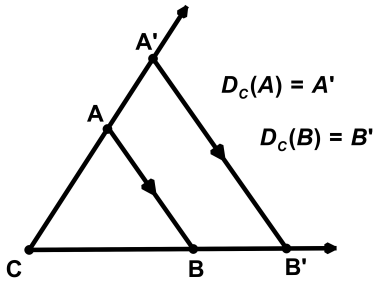
DILATONS AND ANGLES
COMMON CORE GEOMETRY

So far we have seen that dilations, whether in the Euclidean or coordinate plane, map segments to segments with the following properties:

THE TWO PRIMARY PROPERTIES OF DILATIONS

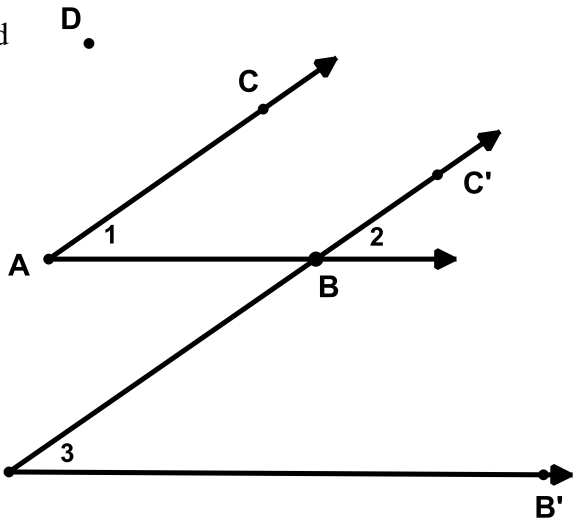
When a dilation of a line segment $\overline{AB}$ not containing the center by a scale factor of k produces $\overline{A'B'}$ then:

1. $A'B' = k \cdot AB$
2. $\overline{A'B'} \parallel \overline{AB}$



In this lesson we are going to see one additional property that is extremely important.

***Exercise* #1:** In the following diagram, $\angle BAC$ has been dilated with a center of D and a scale factor of 2.



(a) List all parallel rays shown in the diagram.

(b) Using your protractor, what are the measures of $\angle 1$ and $\angle 3$?

$m\angle 1 =$ $m\angle 3 =$

(c) Give a reason based on the second property of dilations for why the angle measure doesn't change in this dilation. Refer to the diagram above and use proper terminology regarding parallel lines.

THIRD IMPORTANT PROPERTY OF DILATIONS

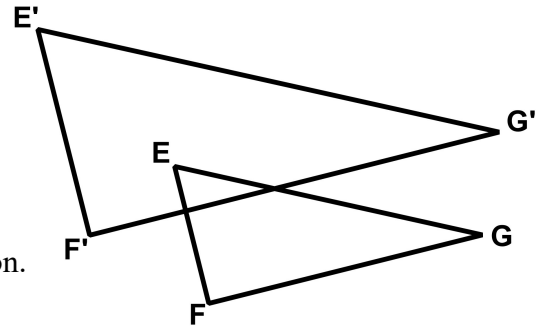
Dilations are **angle preserving**. In other words, the angles of a geometric object do not change when dilated.

***Exercise* #2:** If ΔDEF is dilated by a factor of 5, which of the following statements would be true?

(1) $m\angle D' = 5 \cdot m\angle D$ (3) $m\angle E = \frac{1}{5} \cdot m\angle E'$

(2) $DE = \frac{1}{5} D'E'$ (4) $EF = 5E'F'$

***Exercise* #3:** In the diagram below, $\Delta E'F'G'$ is the image of ΔEFG after a dilation of an unknown scale factor with an unknown center point.



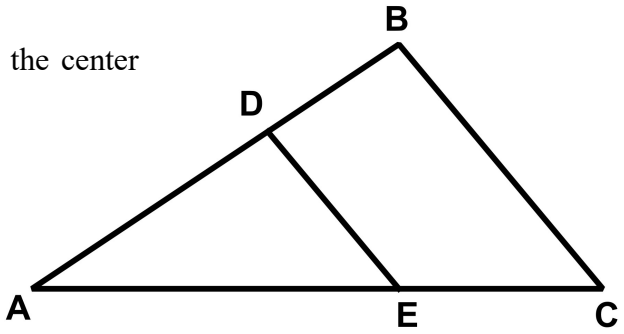
(a) Using tracing paper, verify that the angles of ΔEFG are congruent to the angles of $\Delta E'F'G'$.

(b) Graphically determine the location of the center of the dilation. Mark this point *D*. Leave all construction marks.

(c) Using your ruler to measure lengths, calculate the scale factor to the nearest tenth. Verify it with at least two sets of sides. There may be errors involved due to rounding side lengths.

***Exercise* #4:** In the diagram of ΔABC below, *D* and *E* have been located on $\overline{AB}$ and $\overline{AC}$ such that $\overline{DE} \parallel \overline{BC}$.

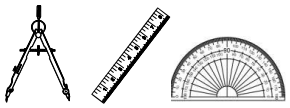
(a) Give a dilation that will map $\overline{BC}$ onto $\overline{DE}$. Specify both the center and scale factor of the dilation. Justify.



(b) If ΔABC was transformed using the dilation specified in (a), explain why its image $\Delta A'B'C'$ would be congruent to ΔADE.

(c) If $AD = 10$, $AB = 15$, $BC = 12$, and $AE = 14$, then algebraically determine the lengths of $\overline{DE}$ and $\overline{AC}$.

Name: _____ Date: _____

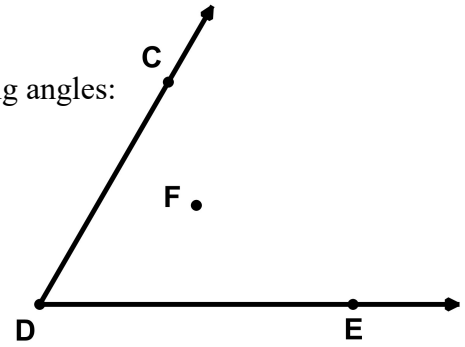


DILATIONS AND ANGLES
COMMON CORE GEOMETRY HOMEWORK

MEASUREMENT AND CONSTRUCTION

1. Given $\angle CDE$ shown below, construct its image, $\angle C'D'E'$ after a dilation using F as the center and a scale factor of 2. Leave all construction marks.

2. Using your protractor, find the measure of both of the following angles:



(a) $m\angle CDE =$

(b) $m\angle C'D'E' =$

3. Why isn't the measure of $\angle C'D'E'$ twice the measure of $\angle CDE$ in #2 above?

PROBLEM SOLVING

4. If rectangle $RSTU$ is dilated with a scale factor of $\frac{1}{2}$ with a center at R then which of the following is the value of $m\angle S'T'U'$?

(1) 45° (3) 120°

(2) 90° (4) 180°

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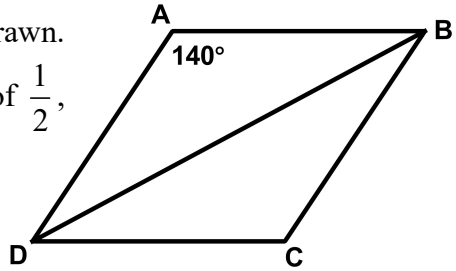
5. In rhombus $ABCD$ shown, $m\angle A = 140^\circ$ and diagonal $\overline{BD}$ has been drawn. If side $\overline{BC}$ is dilated with D as the center and a using a scale factor of $\frac{1}{2}$, then which of the following would be the measure of $\angle DB'C'$?



(1) 10° (3) 40°

(2) 20° (4) 70°

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6. In isosceles triangle ABC it is known that $AB = AC$. If ΔABC is dilated using B as the center point, then which of the following could be an *incorrect* statement? Draw a good diagram!

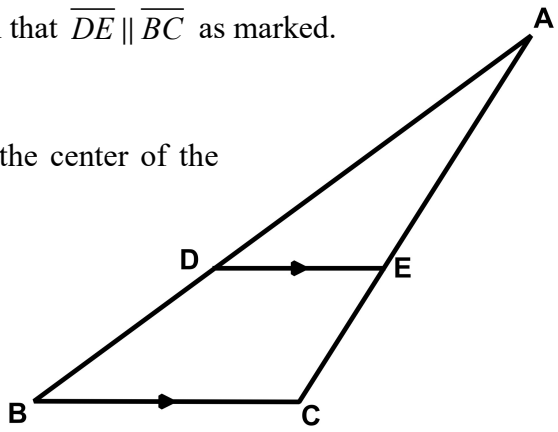
   (1) $\angle BAC \cong \angle BA'C'$

   (2) $\angle ABC \cong \angle A'C'B$

   (3) $\angle C'A'B \cong \angle ABC$

   (4) $\angle BCA \cong \angle A'BC'$

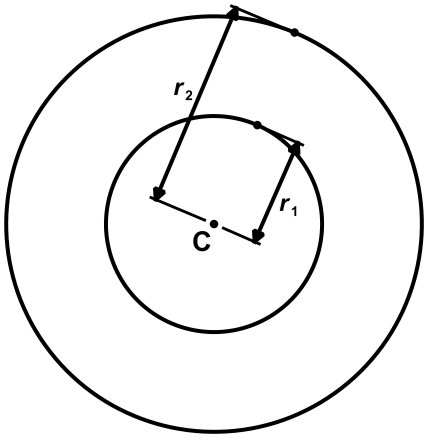
7. In the following diagram, D and E lie on sides $\overline{AB}$ and $\overline{AC}$ such that $\overline{DE} \parallel \overline{BC}$ as marked. It is known that $AD = 18$ and $DB = 12$.



   (a) Give a dilation that would map $\overline{BC}$ onto $\overline{DE}$. State both the center of the dilation and the scaling factor.

   (b) If $BC = 15$ and $AC = 20$, then find the perimeter of ΔADE. Show how you arrived at your answer.

8. Given two concentric circles (circles that have the same center point), with the inner circle having a radius of r_1 and the outer one having a radius of r_2 as shown, give a dilation that would map the inner circle onto the outer circle. Specify both the center of dilation and the scale factor.



REASONING

9. Explain why two perpendicular lines will remain perpendicular if they are both dilated by the same factor and using the same center.