

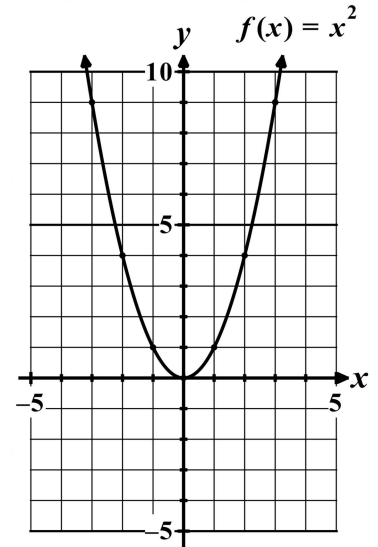
## VERTICAL SHIFTING OF FUNCTIONS

### N-GEN MATH<sup>®</sup> ALGEBRA I



We can build new functions from others by various types of **transformations**. Perhaps the simplest of these transformations is a **vertical shift** of a function.

**Exercise #1:** The function  $f(x) = x^2$  is shown graphed on the grid. The function  $g(x)$  is defined by the formula  $g(x) = f(x) + 3$ .



- Write a formula in terms of  $x$  for the function  $g(x)$ .
- Graph  $g(x)$  on the same grid as  $f(x)$ . Show a table of values in the space to the right.
- Describe how the graph of  $g(x)$  compares to the graph of  $f(x)$ .
- How would the graph of  $h(x) = x^2 - 4$  compare to the graph of  $f(x) = x^2$ ? Produce a graph of  $h(x)$  on the same grid using your answer to this question.

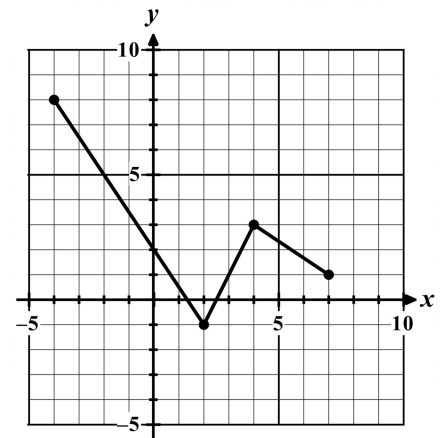
*Vertical shifting of functions is the simplest of all transformations because it occurs by simply adding or subtracting a value from the original function.*

### VERTICAL SHIFTING OF A FUNCTION

If  $k$  is a **positive number** then  $g(x) = f(x) + k$  represents an upward shift of  $f(x)$  by  $k$  units and  $g(x) = f(x) - k$  represents a downward shift of  $f(x)$  by  $k$  units.

**Exercise #2:** The graph of  $f(x)$  is shown. The function  $g(x)$  is defined by the formula  $g(x) = f(x) - 3$

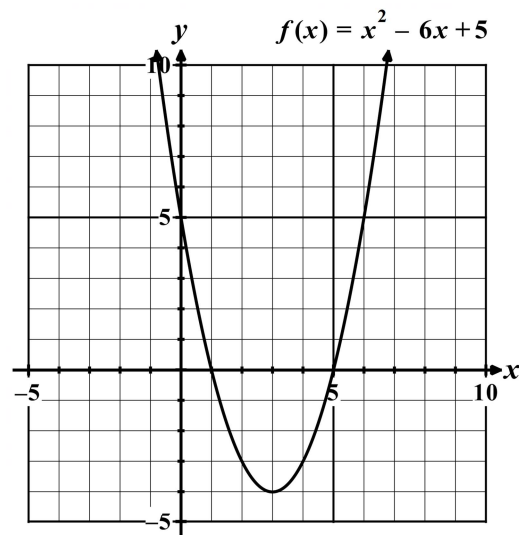
- What is the value of  $g(-4)$ . Show how you found it.
- How will the graph of  $g(x)$  compare to that of  $f(x)$ ?
- Create a graph of  $g(x)$  on the same grid.



Any function can be shifted up or down based on adding or subtracting a constant from its formula.

**Exercise #3:** The function  $f(x) = x^2 - 6x + 5$  is shown graphed on the grid. The function  $g(x)$  is defined by the formula  $g(x) = f(x) + 4$ .

- Give a formula for  $g(x)$  in terms of  $x$ .
- How will the graph of  $g(x)$  compare to that of  $f(x)$ ?
- Using your answer to (b), graph  $g(x)$  on the same grid.
- The function  $g(x)$  has only one zero. Algebraically determine its value.



- How does the graph of  $g(x)$  support your answer to part (d)?

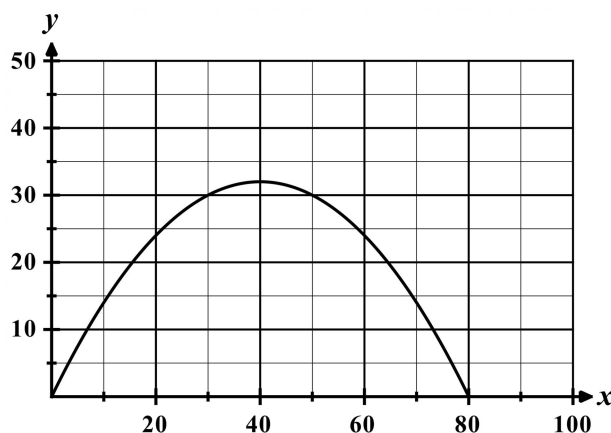
Vertical shifting of functions can also have practical applications.

**Exercise #4:** A physics experiment is run where a projectile is fired from the ground surface. Its height, as a function of the horizontal distance it has traveled, is given by  $h(x) = -0.02x^2 + 1.6x$ . The graph is shown below.

- Using the axis of symmetry formula, show that the maximum of the function  $h(x)$  must occur at  $x = 40$ .
- The projectile is then fired off of the top of a 15-foot-tall platform so that its new height function is given by:

$$g(x) = h(x) + 15$$

Graph  $g(x)$  on the grid



- What would be the peak height the projectile would reach that was fired off the platform? Justify your answer.



**VERTICAL SHIFTING OF FUNCTIONS**  
**N-GEN MATH<sup>®</sup> ALGEBRA I HOMEWORK**

**FLUENCY**

1. If  $f(x) = 3x + 7$  and  $g(x) = f(x) - 9$ , then which of the following is a correct formula for  $g(x)$  in terms of  $x$ ?

(1)  $g(x) = 3x + 16$

(2)  $g(x) = -6x + 7$

(3)  $g(x) = 3x - 2$

(4)  $g(x) = 12x + 7$

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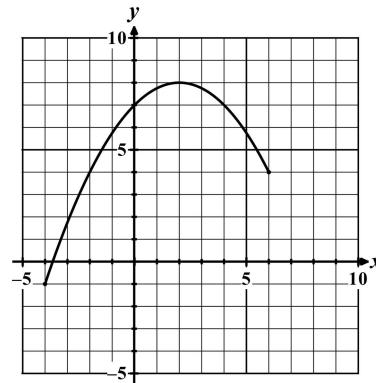
2. The function  $g(x)$  is shown graphed. If  $h(x) = g(x) - 5$  then which of the following is the value of  $h(4)$ ?

(1)  $-6$

(2)  $2$

(3)  $3$

(4)  $8$



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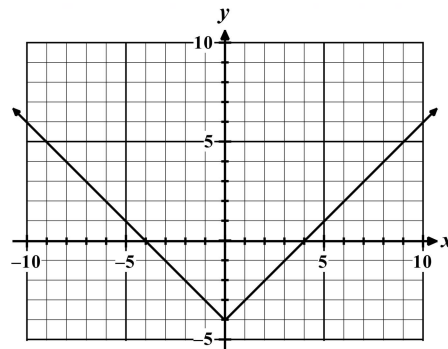
3. Which of the following is the formula for the graph shown below?

(1)  $y = (x - 4)^2$

(2)  $y = x^2 - 4$

(3)  $y = |x - 4|$

(4)  $y = |x| - 4$



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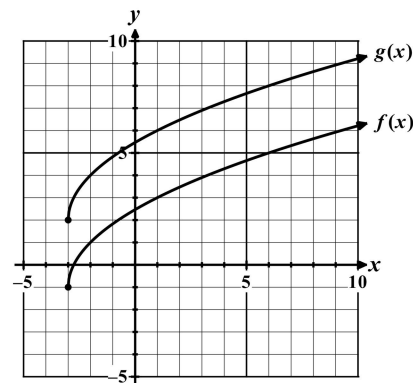
4. The function  $f(x)$  has the formula  $f(x) = 2\sqrt{x+3} - 1$ . Which of the following is the formula for the function  $g(x)$  shown on the same graph?

(1)  $g(x) = 2\sqrt{x+3} + 2$

(2)  $g(x) = 2\sqrt{x+6} - 1$

(3)  $g(x) = 2\sqrt{x+3} + 3$

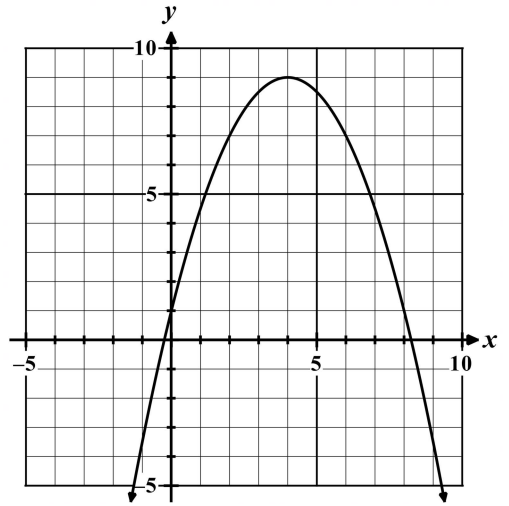
(4)  $g(x) = 2\sqrt{x+6} + 1$



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5. The function  $f(x) = -0.5x^2 + 4x + 1$  shown on the grid. The function  $g(x)$  is defined by  $g(x) = f(x) - 5$ .

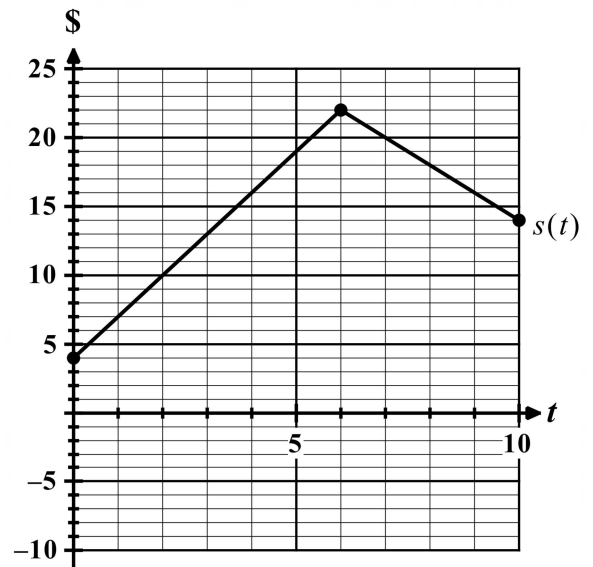


- (a) Give a formula for  $g(x)$  in terms of  $x$ .
- (b) How will the graph of  $g(x)$  compare to the graph of  $f(x)$ ?
- (c) Draw a graph of  $g(x)$  on the grid.
- (d) Give an estimate for the larger solution to the equation  $g(x) = 0$ . Explain how you produced your estimate.

## APPLICATIONS

6. A lemonade stand is operating over a 10-day period. Its sales, in dollars, for a given day since opening is given by the function  $s(t)$  shown graphed below. The lemonade stand must spend \$10 per day on materials. Because of this, the profit they earn is given by the function  $p(t) = s(t) - 10$ .

- (a) What is the value of  $p(4)$ ? Justify.
- (b) Graph  $p(t)$  on the same grid.
- (c) What is the maximum profit the lemonade stand makes?
- (d) For what value of  $t$  is  $p(t) = 0$ ?



- (e) Over what interval of days is the profit at least \$6 per day? Justify.

