

HORIZONTAL STRETCHING OF FUNCTIONS COMMON CORE ALGEBRA I



In the last lesson we saw how multiplying a function by a constant stretched (or compressed) the function's outputs, and thus its graph. This was a **vertical stretch** because it only affected the vertical (output) component of the function for a given input. In today's lesson, we will see what happens to a function when you first manipulate its input.

Exercise #1: The function $f(x)$ is shown on the graph below. Selected points are shown as reference. The function $g(x)$ is defined by $g(x) = f(2x)$. Notice that the multiplication by 2 happens **before** f is even evaluated. This is tricky!

- (a) Find the values of each of the following. Carefully follow the rule for $g(x)$ and show your work.

$$g(2) =$$

$$g(3) =$$

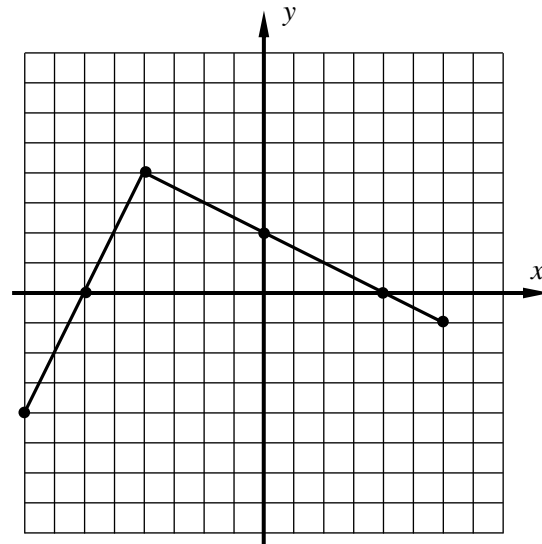
$$g(-2) =$$

$$g(-4) =$$

$$g(0) =$$

$$g(-3) =$$

- (b) Given the definition of $g(x)$, why can we **not** find a value for $g(4)$? Explain.



- (c) State points that must lie on the graph of $g(x)$ based on your work in (a).

- (d) Graph the function $g(x)$ based on your work from (b). Then, state the domain and range of both the original function, $f(x)$ and our new function $g(x)$. What remained the same? What changed?

Original Function: $f(x)$

New Function: $g(x)$

Domain:

Range:

Domain:

Range:

- (e) Describe what happened to the graph of $f(x)$ when we multiplied the function's input by 2.



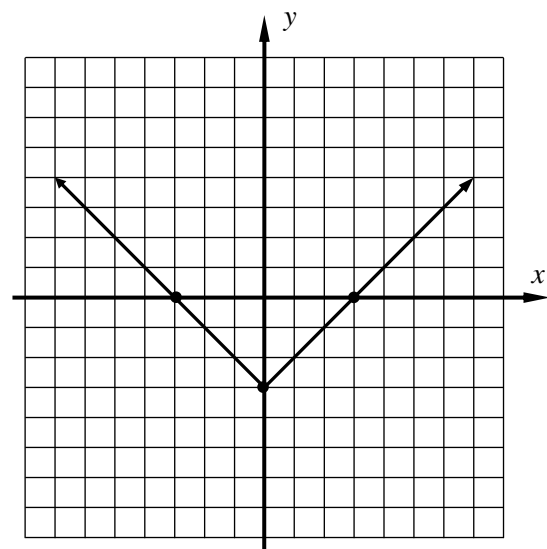
Notice how the **horizontal stretch** worked almost counter to what we would have thought. In other words, when we multiplied the x -value by 2, it **compressed** our graph by a factor of 2. The opposite would also occur.

Exercise #2: The function $f(x) = |x| - 3$ is shown on the graph below. The function $g(x)$ is defined by the

formula $g(x) = \left| \frac{1}{2}x \right| - 3$.

- (a) Use your graphing calculator to produce a table of values for $g(x)$ and graph it on the grid to the right.

| | | | | | | | | | |
|-----|--|--|--|--|--|--|--|--|--|
| x | | | | | | | | | |
| y | | | | | | | | | |



- (b) What was the effect on the graph of $f(x)$ when we multiplied the input by $\frac{1}{2}$?

We can certainly combine the effects of both a vertical stretch and a horizontal stretch. This is harder, but if you can identify the various transformations, then the new function's graph can often be produced from the older function's fairly easily.

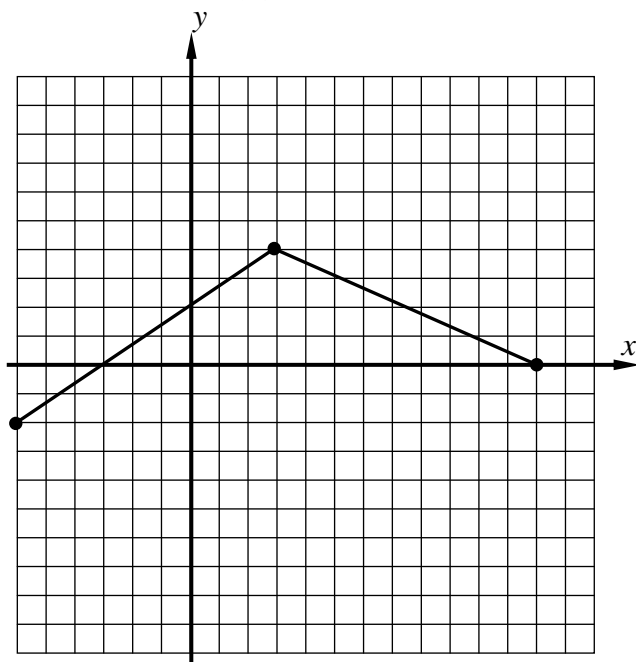
Exercise #3: The graph of $f(x)$ is shown on the grid below. A new function $h(x)$ is defined by:

$$h(x) = 2f(3x)$$

- (a) Evaluate $h(1)$. What point must lie on the graph of $h(x)$ based on this calculation?

- (b) Describe the transformations that must be done to the graph of $f(x)$ to produce the graph of $g(x)$.

- (c) Graph $g(x)$ by plotting the three major points.



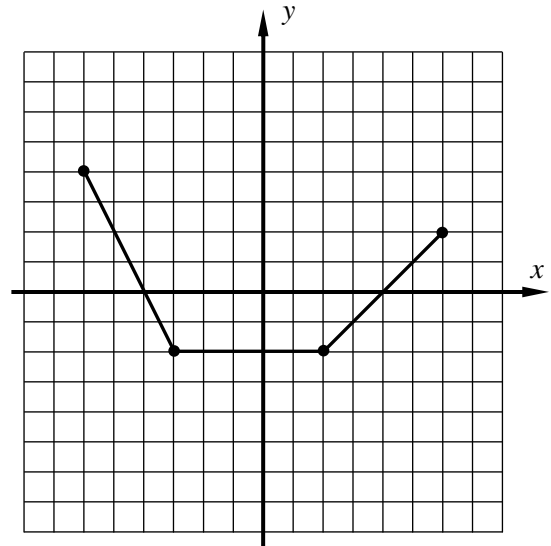
HORIZONTAL STRETCHING OF FUNCTIONS COMMON CORE ALGEBRA I HOMEWORK

FLUENCY

1. The function $f(x)$ is shown graphed on the axes below with selected points highlighted. Two additional functions are defined as:

$$g(x) = f(2x) \quad \text{and} \quad h(x) = 2f(x)$$

Graph both $g(x)$ and $h(x)$ on the same grid and label them.



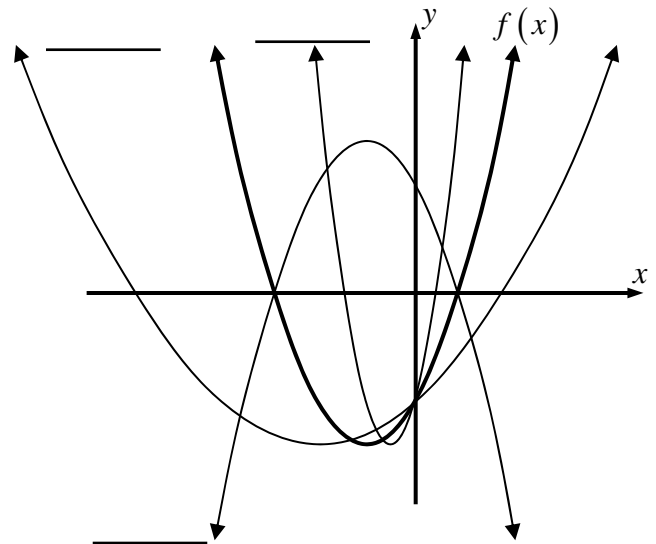
State the domain of $g(x)$ only:

2. The quadratic function $f(x)$ is shown graphed to the right in bold. Three other functions are defined below with equations based on $f(x)$. Label each graph with its appropriate function.

$$g(x) = -f(x)$$

$$h(x) = f(2x)$$

$$k(x) = f\left(\frac{1}{2}x\right)$$



3. Which of the following formulas would indicate that the graph of $h(x)$ was stretched in the horizontal direction by a factor of 3?

(1) $h(3x)$

(3) $h(x) + 3$

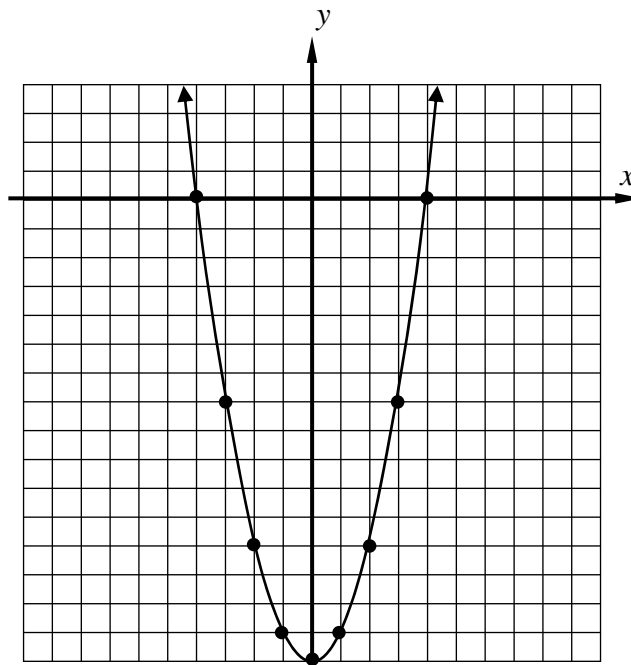
(2) $h\left(\frac{1}{3}x\right)$

(4) $3h(x)$



4. The parabola $f(x) = x^2 - 16$ is shown graphed on the grid below with certain points highlighted. The function $g(x)$ is given by $g(x) = f(2x)$.

- (a) What is the range of the function $f(x)$?
- (b) State the zeroes of $f(x)$.
- (c) The function $g(x)$ will have the equation $g(x) = (2x)^2 - 16$. Using your calculator, create a graph of $g(x)$ on the grid given.
- (d) State the zeroes of $g(x)$. Why does this answer make sense in light of (b)?



REASONING

5. The function $f(x)$ is shown below. Another function is defined by the formula:

$$g(x) = f(2x) + 3$$

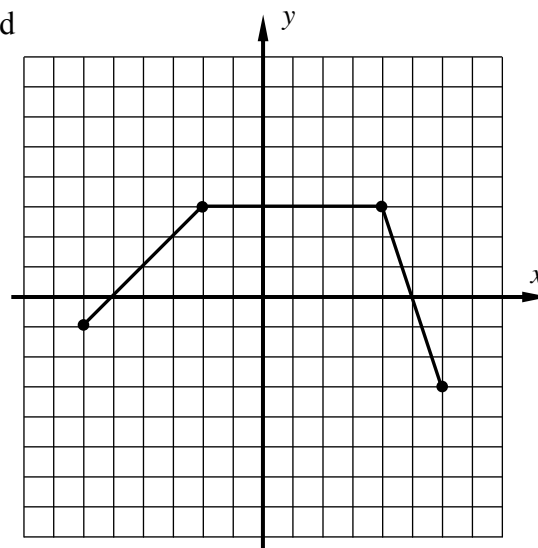
- (a) Evaluate each of the following. Show your work.

$$g(-3) =$$

$$g(-1) =$$

$$g(2) =$$

$$g(3) =$$



- (b) Plot a graph of $g(x)$ based on (a).
- (c) What two transformations occurred to the graph of $f(x)$ to produce the graph of $g(x)$? State them and their order.

