$\qquad$ Date: $\qquad$

## One-to-One Functions Common Core Algebra II



Functions as rules can be divided into various categories based on shared characteristics. One category is comprised of functions known as one-to-one. The following exercise will illustrate the difference between a function that is one-to-one and one that is not.

Exercise \#1: Consider the two simple functions given by the equations $f(x)=2 x$ and $g(x)=x^{2}$.
(a) Map the domain $\{-2,0,2\}$ using each function. Fill in the range and show the mapping arrows.



Range of $g$

(b) What is fundamentally different between these two functions in terms of how the elements of this domain get mapped to the elements of the range?

## One-to-One Functions

A function $f(x)$ is called one-to-one if $x_{1} \neq x_{2}$ implies that $f\left(x_{1}\right) \neq f\left(x_{2}\right)$.
(In other words, different inputs give different outputs.)
Exercise \#2: Of the four tables below, one represents a relationship where $y$ is a one-to-one function of $x$. Determine which it is and explain why the others are not.
(1)

| $x$ | $y$ |
| ---: | ---: |
| 4 | 2 |
| 4 | -2 |
| 9 | 3 |
| 9 | -3 |

(2)

| $x$ | $y$ |
| ---: | ---: |
| -2 | 1 |
| -1 | 0 |
| 0 | 1 |
| 1 | 2 |

(3)

| $x$ | $y$ |
| :---: | :---: |
| 1 | 2 |
| 2 | 4 |
| 3 | 8 |
| 4 | 16 |

(4)

| $x$ | $y$ |
| ---: | ---: |
| -3 | 10 |
| -2 | 9 |
| -1 | 7 |
| -2 | 10 |

Exercise \#3: Consider the following four graphs which show a relationship between the variables $y$ and $x$.

(1)

(2)

(3)

(4)
(a) Circle the two graphs above that are functions. Explain how you know they are functions.
(b) Of the two graphs you circled, which is one-to-one? Explain how you can tell from its graph.

## The Horizontal Line Test

If any given horizontal line passes through the graph of a function at most one time, then that function is one-to-one. This test works because horizontal lines represent constant $y$-values; hence, if a horizontal line intersects a graph more than once, an output has been repeated.

Exercise \#4: Which of the following represents the graph of a one-to-one function?

(1)

(2)

(3)

(4)

Exercise \#5: The distance that a number, $x$, lies from the number 5 on a one-dimensional number line is given by the function $D(x)=|x-5|$. Show by example that $D(x)$ is not a one-to-one function.
$\qquad$

## One-To-One Functions Common Core Algebra II Homework

## Fluency

1. Which of the following graphs illustrates a one-to-one relationship?

(1)

(2)

(3)

(4)
2. Which of the following graphs does not represent that of a one-to-one function?

(1)

(2)

(3)

(4)
3. In which of the following graphs is each input not paired with a unique output?

(1)

(2)

(3)

(4)
4. In which of the following formulas is the variable $y$ a one-to-one function of the variable $x$ ? (Hint - try generating some values either in your head or using TABLES on your calculator.)
(1) $y=x^{2}$
(3) $y=2 x$
(2) $y=|x|$
(4) $y=5$
5. Which of the following tables illustrates a relationship in which $y$ is a one-to-one function of $x$ ?
(1)

| $x$ | $y$ |
| :---: | :---: |
| -2 | -1 |
| 0 | -3 |
| 2 | -1 |
| 4 | 1 |
| 6 | 3 |

(2)

| $x$ | $y$ |
| ---: | ---: |
| -2 | -8 |
| -1 | -1 |
| 0 | 0 |
| 1 | 1 |
| 2 | 8 |

(3)

| $x$ | $y$ |
| ---: | ---: |
| -2 | -5 |
| -1 | -4 |
| 0 | -1 |
| -1 | 7 |
| -2 | 5 |

(4)

| $x$ | $y$ |
| :---: | :---: |
| -2 | 11 |
| -1 | -4 |
| 0 | -5 |
| 1 | -4 |
| 2 | 11 |

## ApPLICATIONS

6. A recent newspaper gave temperature data for various days of the week in table format. In which of the tables below is the reported temperature a one-to-one function of the day of the week?
(1)

| $x$ | $y$ |
| :--- | :---: |
| Mon | 75 |
| Tue | 68 |
| Wed | 65 |
| Thu | 74 |

(2)

| $x$ | $y$ |
| :--- | :---: |
| Mon | 75 |
| Tue | 72 |
| Wed | 68 |
| Thu | 72 |

(3)

| $x$ | $y$ |
| :--- | :---: |
| Mon | 58 |
| Tue | 52 |
| Mon | 81 |
| Tue | 76 |

(4)

| $x$ | $y$ |
| :--- | :---: |
| Mon | 56 |
| Tue | 58 |
| Mon | 85 |
| Tue | 85 |

7. Physics students drop a basketball from 5 feet above the ground and its height is measured each tenth of a second until it stops bouncing. The height of the basketball, $h$, is clearly a function of the time, $t$, since it was dropped.
(a) Sketch the general graph of what you believe this function would look like.
(b) Is the height of the ball a one-to-one function of time? Explain your answer.


## ReAsoning

8. Consider the function $f(x)=\operatorname{round}(x)$, which rounds the input, $x$, to the nearest integer. Is this function one-to-one? Explain or justify your answer.
