Name: ___________________________________ Date: _________________

Congruence Reasoning About Triangles
Common Core Geometry

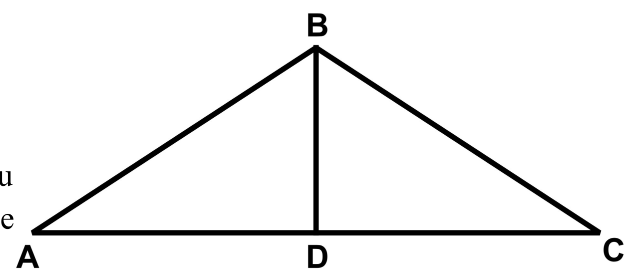
Two triangles will be **congruent** if **they have the same size and shape**. We can now say, given our studies of rigid motions, that:

> **Triangle Congruence**
>
> Two triangles in the plane are **congruent** if a sequence of rigid motions can be found that make the vertices of one triangle coincide with the vertices of the other (or lie on top of each other).

***Exercise* #1:** In the diagram below we know the following:

Given: $\overline{ADC}$, $\overline{BD} \perp \overline{AC}$ and $\overline{AD} \cong \overline{CD}$.

Prove: $\Delta BDC \cong \Delta BDA$



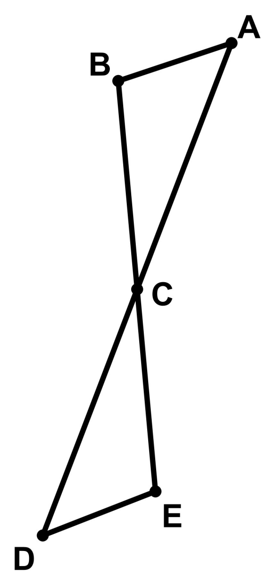
(a) Based on the givens and the diagram, what rigid motion do you think will be used to map ΔBDC onto ΔBDA? Try the transformation with tracing paper.

(b) Prove (using rigid motions): $\Delta BDC \cong \Delta BDA$.

***Exercise* #2:** In the diagram below we know the following:

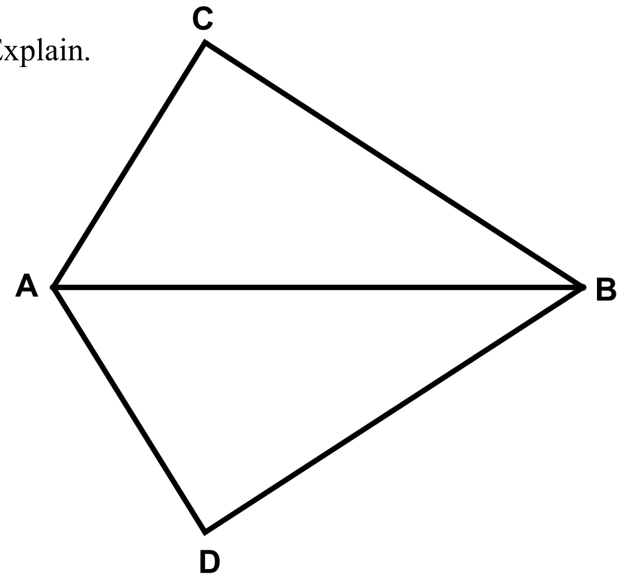
Given: $\overline{ACD}$, $\overline{BCE}$ with C being the midpoint of both $\overline{AD}$ and $\overline{BE}$.

Prove (using rigid motions): $\Delta ABC \cong \Delta DEC$



These transformational geometry proofs can be challenging, even in fairly simple situations.

***Exercise* #3:** In the following diagram it is known that $\overline{AB}$ bisects both $\angle CAD$ and $\angle CBD$. This will be enough, using rigid motions, to show that $\Delta ACB \cong \Delta ADB$.



(a) What angle pairs must be congruent based on what is known? Explain. Mark these pairs on the diagram.

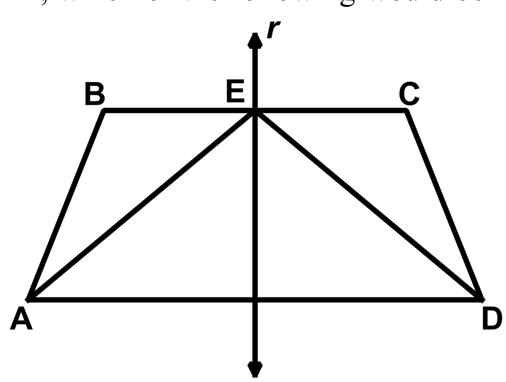
(b) If ΔADB was reflected in $\overline{AB}$, what two points would not move (called "fixed points")? Why wouldn't they?

(c) If ΔADB was reflected in $\overline{AB}$, why must the image of point D, i.e. D', fall on both **rays** $\overrightarrow{AC}$ and $\overrightarrow{BC}$?

(d) Since D' must fall on both $\overrightarrow{AC}$ and $\overrightarrow{BC}$, where does this tell you that it must lie? Explain.

(e) Explain why do we now know that $\Delta ACB \cong \Delta ADB$

***Exercise* #4:** Given that line r is the perpendicular bisector of $\overline{BC}$ and $\overline{AD}$, which of the following would be used to justify that ΔABE is congruent to ΔDCE?



(1) a reflection of ΔABE across the line $\overleftrightarrow{AD}$

(2) a 180° rotation of ΔABE about point E

(3) a translation of ΔABE in the direction of $\overrightarrow{BC}$ by a distance of BE.

(4) a reflection of ΔABE across line r.

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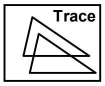
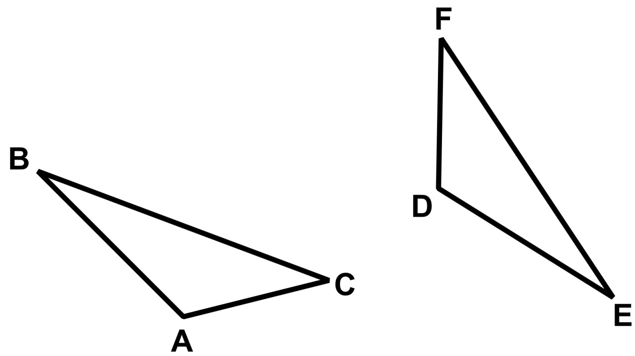


CONGRUENCE REASONING ABOUT TRIANGLES
COMMON CORE GEOMETRY HOMEWORK

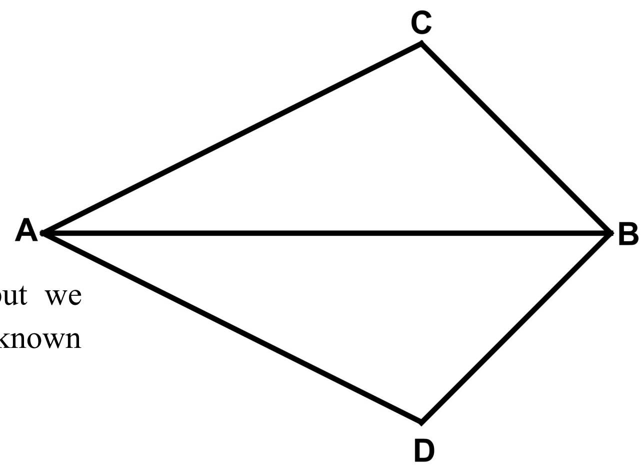
MEASUREMENT AND CONSTRUCTION

1. Are ΔABC and ΔDEF pictured below congruent? Use tracing paper to decide.



REASONING

2. In the diagram shown the following is known:



Given: $\overline{AB}$ bisect $\angle CBD$ and $BD = BC$.

Prove: $\Delta ADB \cong \Delta ACB$ using rigid motion reasoning

(a) Explain why we know that $m\angle CBA = m\angle DBA$ but we **don't** know that $m\angle CAB = m\angle DAB$. Mark the known equal angles.

(b) A reflection of ΔABD across $\overline{AB}$ leaves what points unmoved ("fixed")? Explain why they don't move.

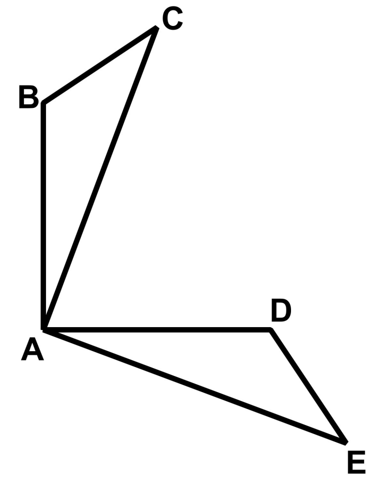
(c) The image of D under the reflection in $\overline{AB}$ must fall on C. This needs both $m\angle CBA = m\angle DBA$ and $BD = BC$. Explain why.

(d) Explain why this reflection in $\overline{AB}$ now shows that $\Delta ADB \cong \Delta ACB$.

3. In the following diagram ΔABC and ΔADE are given with the following known:

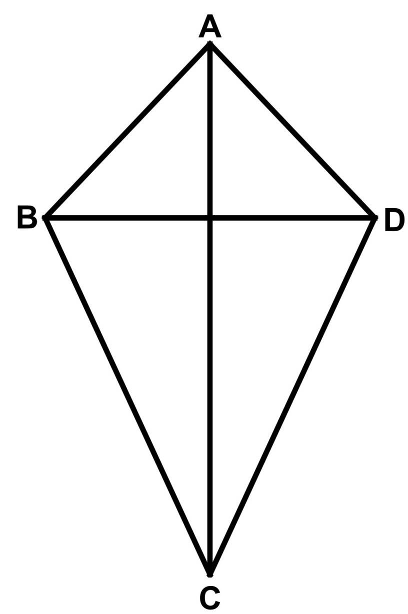
Given: $\overline{AB} \perp \overline{AD}$, $\overline{AC} \perp \overline{AE}$, $AB = AD$, and $AC = AE$

Give a rigid motion transformation that shows that $\Delta ABC \cong \Delta ADE$. Give both the rigid motion and explain why it maps ΔABC onto ΔADE.



4. In the following diagram ΔABC and ΔADC are drawn with a shared side of $\overline{AC}$. We are given the following information:

Given: $AB = AD$ and $CB = CD$.



(a) Point A is equidistant from points B and D based on the givens. So is point C. That means that $\overline{AC}$ has what special relationship to segment $\overline{BD}$? Explain. (See Unit #2.Lesson #4)

(b) If we reflect ΔABC across $\overline{AC}$ then what points stay fixed? Why?

(c) Why must point B fall on D after a reflection in $\overline{AC}$? (See part a).

(d) Why does a reflection of ΔABC in $\overline{AC}$ show that $\Delta ABC \cong \Delta ADC$?