Name: _____ Date: _____

SOLVING EQUATIONS USING IDENTITIES
ALGEBRA 2 WITH TRIGONOMETRY

Some special types of trigonometric equations can only be solved algebraically by substitution of an appropriate trigonometric identity. Since an identity is a true equality for all values of the variable, the principle of mathematical substitution allows us to interchange these equal quantities. The main identities that you will need are as follows. Note that the Pythagorean Identity should be memorized.

TRIGONOMETRIC IDENTITIES FOR EQUATION SOLVING

$\sin^2 A + \cos^2 A = 1$

$\sin(2A) = 2\sin A \cos A$

$\cos(2A) = 2\cos^2 A - 1$

$\cos(2A) = 1 - 2\sin^2 A$

***Exercise* #1:** Solve the equation $\sin(2A) - \cos A = 0$ for all values of A on the interval $0^\circ \le A \le 360^\circ$.

When a substitution of $\sin(2A)$ is needed, the choice is straightforward, because there is only one identity. It is more difficult when a choice must be made for $\cos(2A)$. In general, if you can avoid mixing sines and cosines upon substitution, do so.

***Exercise* #2:** Solve the equation $\cos(2A) + 5\cos A + 3 = 0$ for all values of A on the interval $0^\circ \le A \le 360^\circ$.

The **Pythagorean Identity** is particularly useful when the quadratic term of an equation needs to be replaced.

***Exercise* #3:** Solve the equation $\cos^2 x + 4\sin x + 4 = 0$ for all values of x on the interval $0^\circ \le x \le 360^\circ$.

Sometimes the substitution is used to create an incomplete quadratic equation, as seen in the next exercise.

***Exercise* #4:** Solve the equation $3\cos(2\theta) = 2$ for all values of θ on the interval $0^\circ \le \theta \le 360^\circ$. Express all answers to the nearest *tenth* of a degree. Note that there are multiple ways to solve this problem.

***Exercise* #5:** Show that the trigonometric equation $\cos(2\alpha) + 3\sin^2 \alpha - 6\sin\alpha + 7 = 0$ does not have any solutions.

Name: _____ Date: _____

SOLVING EQUATIONS USING IDENTITIES
ALGEBRA 2 WITH TRIGONOMETRY - HOMEWORK

TRIGONOMETRIC IDENTITIES FOR EQUATION SOLVING

$\sin^2 A + \cos^2 A = 1$

$\cos(2A) = 2\cos^2 A - 1$

$\sin(2A) = 2\sin A \cos A$

$\cos(2A) = 1 - 2\sin^2 A$

SKILLS

1. Find all values of θ on the interval $0^\circ \le \theta \le 360^\circ$ that solve $\sin(2\theta) = \sin\theta$.

2. Determine all values of x on the interval $0^\circ \le x \le 360^\circ$ that satisfy $\cos(2x) = 0$.

3. Solve the equation $\cos(2A) - 3\sin A - 2 = 0$ for all values of A on $0^\circ \le A \le 360^\circ$.

4. Solve the equation $4\sin(2\theta) - 3\cos\theta = 0$ for all values on the interval $0^\circ \le \theta \le 360^\circ$. Express any non-integer answers to the nearest *tenth* of a degree.

5. Find all values of x over the interval $0^\circ \le x \le 360^\circ$ that solve $3\sin^2 x - 5\cos x - 1 = 0$. Express all answers accurate to the nearest *tenth* of a degree.

6. Solve the equation $5\cos(2\alpha) + 3 = 0$ for all values of α on the interval $0^\circ \le \alpha \le 360^\circ$. Round all answers to the nearest *tenth* of a degree.

7. Find all solutions to $\cos(2\beta) - 7\cos\beta + 6 = 0$ over the interval $0^\circ \le \beta \le 360^\circ$.