

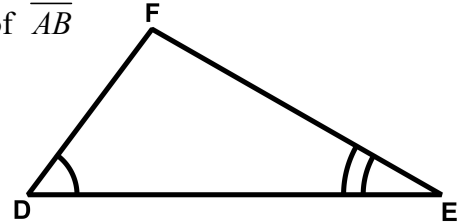
**SIMILARITY CRITERIA**  
**COMMON CORE GEOMETRY**



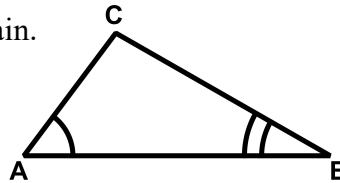
We saw in the last lesson that if two triangles are similar then **corresponding pairs of angles** have **equal measures** and **corresponding pairs of sides** have **proportional lengths**. Just as with **congruence**, we can tell that two triangles are similar with less information than all angle and side pairs. There are three major criteria that can be used to prove triangles are congruent. We will explore each one using **similarity transformations**.

**Exercise #1:** Given  $\triangle ABC$  and  $\triangle DEF$  with  $\angle A \cong \angle D$  and  $\angle B \cong \angle E$ , do the following:

- (a) Give a dilation of  $\triangle ABC$  centered at  $A$  that would make the image of  $\overline{AB}$  congruent to  $\overline{DE}$ . Be as specific as possible.



- (b) Would this dilation change the  $m\angle A$  or  $m\angle B$ ? Explain.

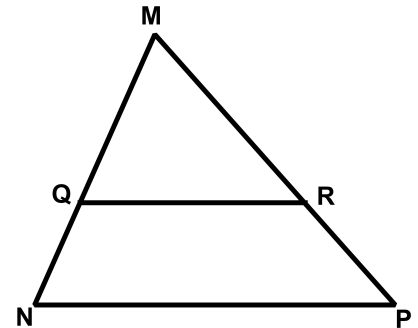


- (c) Why would  $\triangle A'B'C'$  now be congruent to  $\triangle DEF$ ?

**THE ANGLE-ANGLE CRITERIA FOR SIMILARITY**

If two angles of one triangle are congruent to two angles of another triangle, the two triangles are similar.

**Exercise #2:** In the diagram shown,  $\overline{QR} \parallel \overline{NP}$ . Prove that  $\triangle MQR$  is similar to  $\triangle MNP$ .



**Exercise #3:** If in the above diagram  $MQ = 8$ ,  $NQ = 6$  and  $NP = 21$ , then what is the length of  $\overline{QR}$ ?



There are two additional criteria for similarity that we will present and then informally prove.

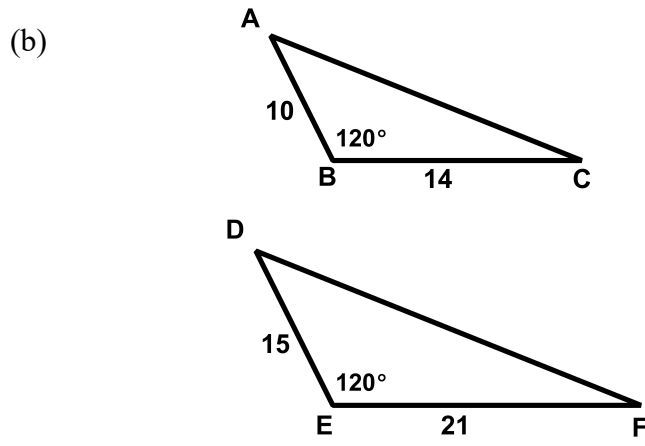
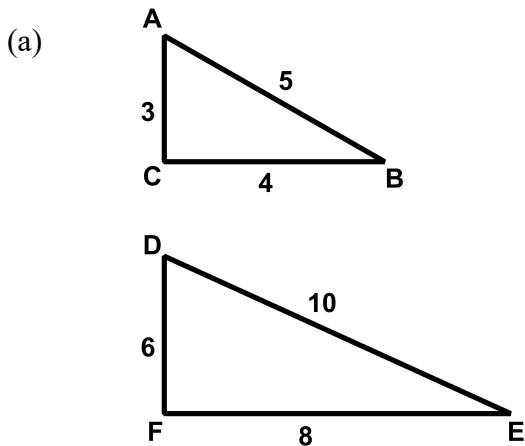
**THE SIDE-SIDE-SIDE CRITERIA FOR SIMILARITY**

If all three pairs of corresponding sides of two triangles are proportional, then the two triangles are similar.

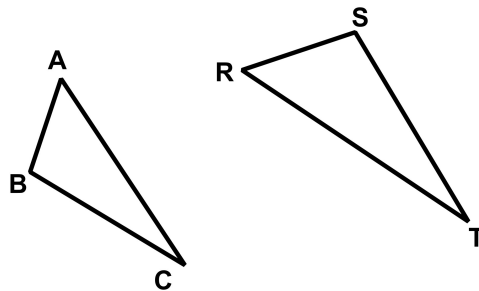
**THE SIDE-ANGLE-SIDE CRITERIA FOR SIMILARITY**

If two pairs of sides of two triangles are proportional and the angles included between the proportional side pairs are congruent, then the two triangles are similar.

**Exercise #4:** Say that you have the following diagrams with only the information marked. In both cases, give a dilation, with a center and factor, that would map  $\triangle ABC$  onto an image  $\triangle A'B'C'$  that is congruent to  $\triangle DEF$ . Justify the congruence using a congruence theorem.

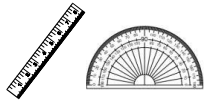


**Exercise #5:** In the diagram below, which of the following information would not be sufficient to show that  $\triangle ABC$  is similar to  $\triangle RST$ ?



- (1)  $\frac{RS}{AB} = \frac{ST}{BC} = \frac{TR}{CA}$
- (2)  $\frac{AB}{RS} = \frac{AC}{RT}$  and  $m\angle A = m\angle R$
- (3)  $m\angle S = m\angle B$  and  $m\angle C = m\angle T$
- (4)  $\frac{RT}{AC} = \frac{TS}{CB}$  and  $m\angle B = m\angle S$

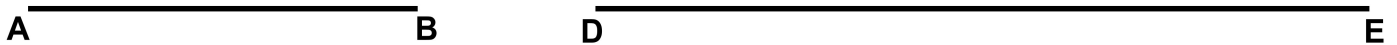




**SIMILARITY CRITERIA**  
**COMMON CORE GEOMETRY HOMEWORK**

**MEASUREMENT AND CONSTRUCTION**

1. In the diagram below, two segments have been drawn, where  $DE = 2AB$ . Using your protractor, create two triangles,  $\triangle ABC$  and  $\triangle DEF$ , from these bases such that  $m\angle A = m\angle D = 30^\circ$  and  $m\angle B = m\angle E = 60^\circ$



2. In the above diagram, measure  $\overline{AC}$ ,  $\overline{BC}$ ,  $\overline{DF}$  and  $\overline{EF}$  to the nearest millimeter. Then, calculate the ratios asked for.

$$AC = \qquad \qquad \qquad DF = \qquad \qquad \qquad \frac{DF}{AC} =$$

$$BC = \qquad \qquad \qquad EF = \qquad \qquad \qquad \frac{EF}{BC} =$$

3. Why were the ratios you calculated in #2 equal (or they should have)?

**PROBLEM SOLVING**

4. Which of the following would be enough information to know that  $\triangle ABC \sim \triangle MNP$ ?

(1)  $m\angle P = m\angle C$  and  $\frac{BC}{NP} = \frac{AB}{MN}$

(2)  $m\angle A = m\angle M$  and  $\frac{AB}{MN} = \frac{AC}{MP}$

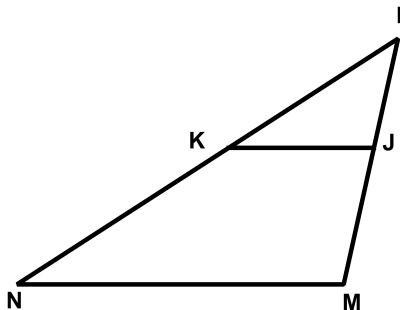
(3)  $m\angle B = m\angle N$  and  $AB = MN$

(4)  $m\angle P = m\angle C$  and  $\frac{BC}{NP} = \frac{MP}{AC}$



5. Which of the following measurements below would justify that  $\triangle LJK \sim \triangle LMN$  ?

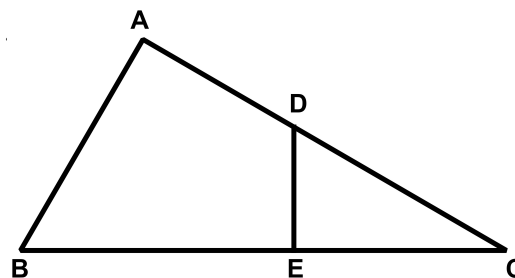
- (1)  $KJ = 8$ ,  $NM = 16$ ,  $KL = 11$ , and  $LN = 22$
- (2)  $KL = 8$ ,  $LN = 12$ ,  $LJ = 6$ , and  $LM = 9$
- (3)  $KJ = 3$ ,  $NM = 9$ ,  $LJ = 4$ , and  $JM = 8$
- (4)  $KL = 12$ ,  $LN = 20$ ,  $LJ = 9$ , and  $JM = 15$



**REASONING**

6. In the following diagram it is known that  $\overline{AB} \perp \overline{AC}$  and  $\overline{DE} \perp \overline{BC}$ .

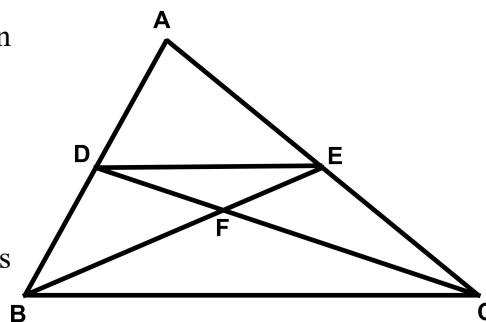
(a) Explain why  $\triangle BAC$  must be similar to  $\triangle DEC$ .



(b) If  $BC = 20$ ,  $DE = 4$ , and  $DC = 8$  then find the length of  $\overline{AB}$ . Show your work.

7. In the diagram of  $\triangle ABC$  shown below, the midpoints of sides  $\overline{AB}$  and  $\overline{AC}$  are  $D$  and  $E$  respectively. Segments  $\overline{BE}$  and  $\overline{CD}$  have been drawn and intersect at  $F$ .

(a) Because  $D$  and  $E$  are midpoints of sides  $\overline{AB}$  and  $\overline{AC}$ , what can we conclude about  $\overline{DE}$  and  $\overline{BC}$ ? (See Unit #6.Lesson #4)?



(b) What similarity criteria could be used to prove that  $\triangle DEF$  is similar to  $\triangle BCF$ ? Explain.

(c) Explain why  $\frac{BF}{FE} = 2$ .

