Name: ____________________________________ Date: __________________

INVERSE FUNCTIONS
COMMON CORE ALGEBRA II

The idea of inverses, or opposites, is very important in mathematics. So important, in fact, that the word is used in many different contexts, including the additive and multiplicative inverses of a number. The actions of certain functions can be reversed as well. The rules governing the reversal themselves can be functions.

***Exercise* #1:** Consider the two linear functions given by the formulas $f(x)=\frac{3x+7}{2}$ and $g(x)=\frac{2x-7}{3}$.

(a) Calculate $f(5)$ and $g(11)$.

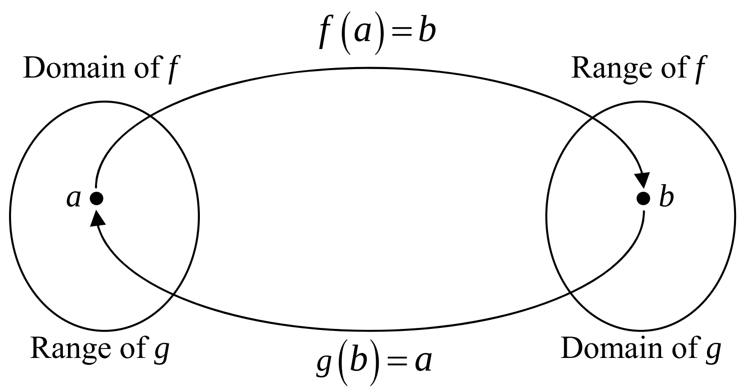
(b) Calculate $f(0)$ and $g\left(\frac{7}{2}\right)$.

(c) Calculate $f(g(-1))$.

(d) Calculate $f(g(5))$.

(e) Without calculation, determine the value of $f(g(\pi))$.

The two functions seen in Exercise #1 are inverses because they literally "undo" one another. The general idea of inverses, $f(x)$ and $g(x)$, is shown below in the mapping diagram.



***Exercise* #2:** If the point $(-3, 5)$ lies on the graph of $y = f(x)$, then which of the following points must lie on the graph of its inverse?

(1) $(3,-5)$

(2) $(-5,3)$

(3) $(5,-3)$

(4) $\left(-\frac{1}{3},\frac{1}{5}\right)$

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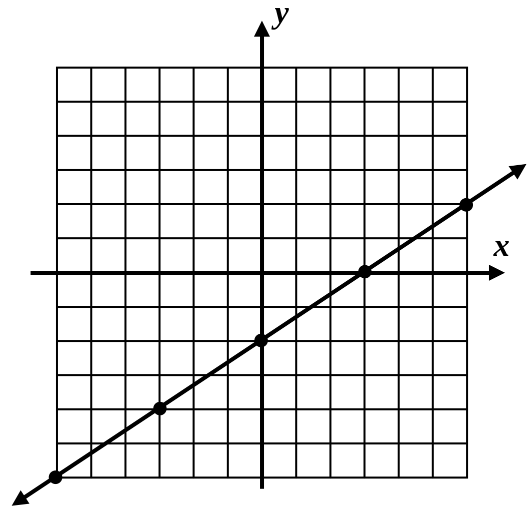
Inverse functions have their own special notation. It is shown in the box below.

INVERSE FUNCTION NOTATION

If a function $y = f(x)$ has an inverse that is also a function we represent it as $y = f^{-1}(x)$.

***Exercise* #3:** The linear function $f(x) = \frac{2}{3}x - 2$ is shown graphed below. Use its graph to answer the following questions.



(a) Evaluate $f^{-1}(2)$ and $f^{-1}(-4)$.

(b) Determine the *y*-intercept of $f^{-1}(x)$.

(c) On the same set of axes, draw a graph of $y = f^{-1}(x)$.

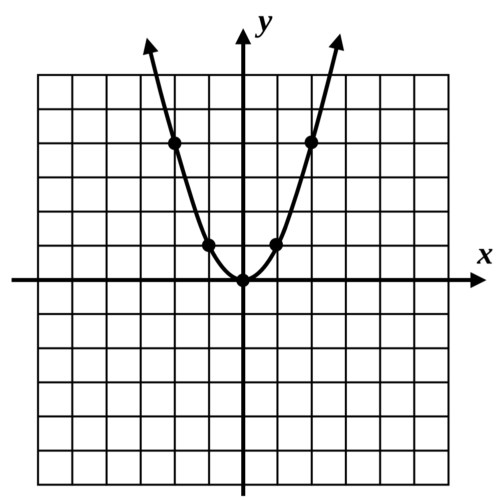
***Exercise* #4:** A table of values for the simple quadratic function $f(x) = x^2$ is given below along with its graph.

x	-2	-1	0	1	2
$f(x)$	4	1	0	1	4

(a) Graph the inverse by switching the ordered pairs.

x					
$f^{-1}(x)$					

(b) What do you notice about the graph of this function's inverse?



EXISTENCE OF INVERSE FUNCTIONS

A function will have an inverse that is also a function if and only if it is one-to-one. Hence, a quick way to know if a function has an inverse that is also a function is to apply the Horizontal Line Test.

Name: ____________________________________ Date: _________________

INVERSE FUNCTIONS
COMMON CORE ALGEBRA II HOMEWORK

FLUENCY

1. If the point $(-7, 5)$ lies on the graph of $y = f(x)$, which of the following points must lie on the graph of its inverse?

(1) $(5, -7)$ (3) $(7, -5)$

(2) $\left(-\frac{1}{7}, \frac{1}{5}\right)$ (4) $\left(\frac{1}{7}, -\frac{1}{5}\right)$ __________

2. The function $y = f(x)$ has an inverse function $y = f^{-1}(x)$. If $f(a) = -b$ then which of the following must be true?

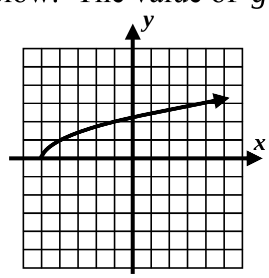
(1) $f^{-1}(-b) = -a$ (3) $f^{-1}(-b) = a$

(2) $f^{-1}\left(\frac{1}{a}\right) = -\frac{1}{b}$ (4) $f^{-1}(b) = -a$ __________

3. The graph of the function $y = g(x)$ is shown below. The value of $g^{-1}(2)$ is

(1) 2.5 (3) 0.4

(2) −4 (4) −1



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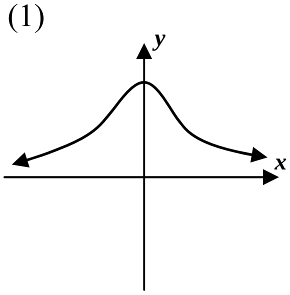
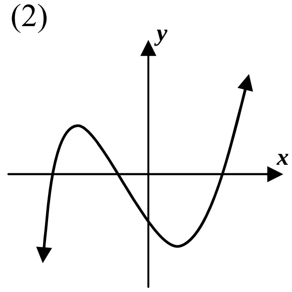
4. Which of the following functions would have an inverse that is also a function?

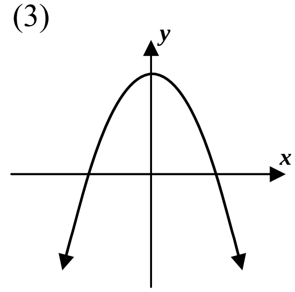
(1)



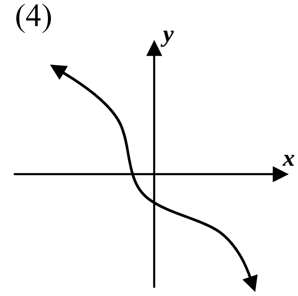
(2)



(3)



(4)



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5. For a one-to-one function it is known that $f(0) = 6$ and $f(8) = 0$. Which of the following must be true about the graph of this function's inverse?

(1) its y-intercept = 6 (3) its x-intercept = −6

(2) its y-intercept = 8 (4) its x-intercept = −8

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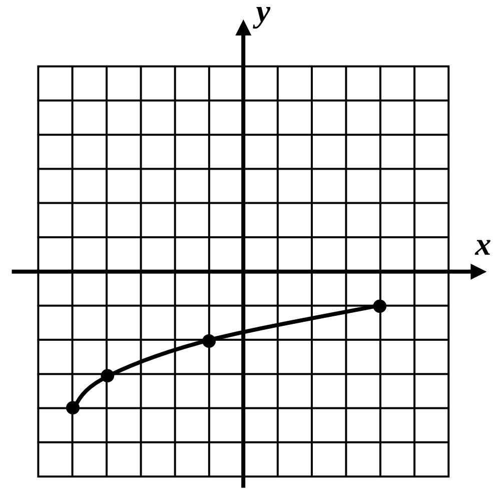
6. The function $y = h(x)$ is entirely defined by the graph shown below.

(a) Sketch a graph of $y = h^{-1}(x)$. Create a table of values if needed.

(b) Write the domain and range of $y = h(x)$ and $y = h^{-1}(x)$ using interval notation.



$y = h(x)$

Domain:

Range:

$y = h^{-1}(x)$

Domain:

Range:

APPLICATIONS

7. The function $y = A(r) = \pi r^2$ is a one-to-one function that uses a circle's radius as an input and gives the circle's area as its output. Selected values of this function are shown in the table below.

r	1	2	3	4	5	6
$A(r)$	π	4π	9π	16π	25π	36π

(a) Determine the values of $A^{-1}(9\pi)$ and $A^{-1}(36\pi)$ from using the table.

(b) Determine the values of $A^{-1}(100\pi)$ and $A^{-1}(225\pi)$.

(c) The original function $y = A(r)$ converted an input, the circle's radius, to an output, the circle's area. What are the inputs and outputs of the inverse function?

Input:

Output:

REASONING

8. The domain and range of a one-to-one function, $y = f(x)$, are given below in set-builder notation. Give the domain and range of this function's inverse also in set-builder notation.

$y = f(x)$

Domain: $\{x \mid -3 \le x < 5\}$

Range: $\{y \mid y > -2\}$

$y = f^{-1}(x)$

Domain:

Range: