SOLVING EQUATIONS WITH VARIABLES ON BOTH SIDES
N-GEN MATH® 8

Equations are just statements about the equality of two expressions. The two expressions form the two “sides” of the equation and are always separated by the equality sign. So far, we have only worked with solving equations that have variables on one side. But today we begin to work with ones that have variables on both sides.

Exercise #1: Consider the equation $5x + 6 = 2x + 30$.

(a) Show that $x = 5$ is not a solution to this equation.

(b) Show that $x = 8$ is a solution to this equation.

When we have solved two-step linear equations previously, we have been applying properties of equality in order to isolate the variable (get it by itself). Here are those properties again.

**Properties of Equality**

1. **Addition/subtraction property**: If $a = b$ then $a + c = b + c$ and $a - c = b - c$
2. **Multiplication/division property**: If $a = b$ then $c \cdot a = c \cdot b$ and $\frac{a}{c} = \frac{b}{c}$ (as long as $c \neq 0$).

We want to use these properties when variables are on both sides of the equation. They give us permission to add, subtract, multiply and divide both sides of the equation in order to isolate the variable.

Exercise #2: Let’s revisit the equation $5x + 6 = 2x + 30$. We will now isolate the variable.

(a) The first two steps would be the following:

$$
\begin{align*}
5x + 6 &= 2x + 30 \\
-6 &= -6
\end{align*}
$$

$$
\begin{align*}
5x &= 2x + 24 \\
-2x &= -2x
\end{align*}
$$

$$
3x = 24
$$

(b) Solve the simplified equation below:

$$
3x = 24
$$

What property did you use this time?

What property justifies both steps?
Learning to solve simple equations with variables on both sides is the next evolution in your equation solving tool bag. The next exercise gives you practice with all sorts of types.

**Exercise #3:** Solve each equation shown below. Show each step. Your answers will all be integers but could be negative as well as positive.

(a) \(7n + 5 = 2n + 25\)  
(b) \(8x - 12 = 3x + 33\)  
(c) \(9y + 5 = 2y - 37\)

(d) \(3t - 12 = -2t + 43\)  
(e) \(2c - 15 = 7c + 25\)  
(f) \(-2k + 10 = k - 35\)

Many equations that you solve will have variables on both sides. Most often, students feel comfortable isolating the variable on the left side of the equation. Sometimes it is easiest to deal with negative and positive numbers by “flipping” the equality.

**Exercise #4:** Equality is symmetric, meaning if \(a = b\) then \(b = a\). Rewrite each of the following equations by “flipping” its two sides and then solving as before.

(a) \(2x + 45 = 7x + 5\)  
(b) \(5t + 36 = 8t\)  
(c) \(-2m - 9 = 4m + 3\)
SOLVING EQUATIONS WITH VARIABLES ON BOTH SIDES
N-GEN MATH® 8 HOMEWORK

FLUENCY

1. A true equation is like a balanced scale. In the diagram, the equation:

   \[ 4x + 2 = 2x + 8 \]

   is shown, where \( x \) represents some unknown number of blocks.

   (a) As your first step, subtract 2 from both sides of this equation while crossing two blocks off from both sides of the scale.

   (b) As a second set, subtract \( 2x \) from both sides of the equation and cross two \( x \)'s off from both sides of the scale.

   (c) Finish solving the equation.

2. Which of the following is the solution to the equation \( 8x + 7 = 2x + 37 \)?

   (1) \( x = 5 \)
   (2) \( x = 11 \)
   (3) \( x = 3 \)
   (4) \( x = 9 \)

3. Which value of \( a \) below solves the equation: \( 3a + 1 = -a - 3 \)?

   (1) \( -6 \)
   (2) \( 8 \)
   (3) \( -1 \)
   (4) \( 5 \)
4. Solve each of the following equations. The solutions will all be integers (no fractions) but will include both negative and positive answers (one answer will even be zero).

(a) $9w + 4 = 2w + 32$  
(b) $6x - 5 = 2x + 7$  
(c) $8 + 4 = 3t - 6$

(d) $10y + 7 = 4y + 7$  
(e) $3z - 2 = 5z + 20$  
(f) $-4k + 10 = -7k + 55$

(g) $2x + 45 = 7x$  
(h) $5 - y = y + 25$  
(i) $-2n + 15 = n - 60$

**REASONING**

5. Consider the equation $x + 2 = x + 8$. What happens when you try to solve this equation? What do you think this tells you about it? (We will discuss these “types” of equations more in a later lesson.)