Name: ___________________________________ Date: _________________

INSCRIBING REGULAR POLYGONS
COMMON CORE GEOMETRY

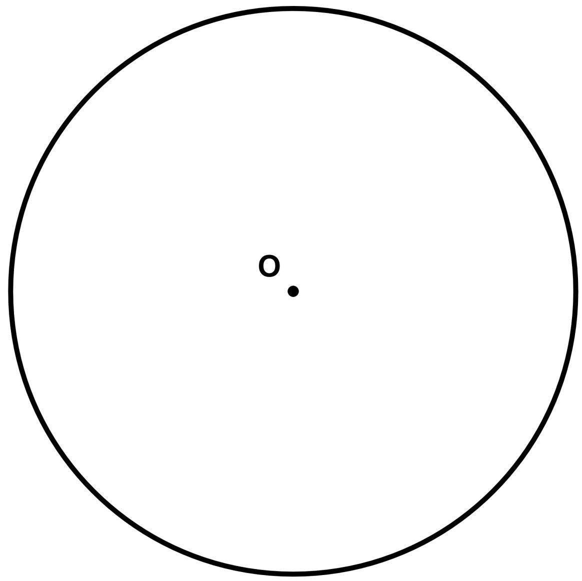
In our last set of constructions, at least for now, we will see how to inscribe a square, equilateral triangle, and regular hexagon in a circle using only a compass and straightedge. We first begin with the square.

***Exercise* #1:** A circle with center point O is shown below. Do the following.

(a) Construct (draw) a diameter of the circle in any direction. Mark its endpoints A and B.

(b) Construct the perpendicular bisector of $\overline{AB}$. Mark its endpoints C and D.

(c) Draw polygon $ACBD$. This polygon will be a square.



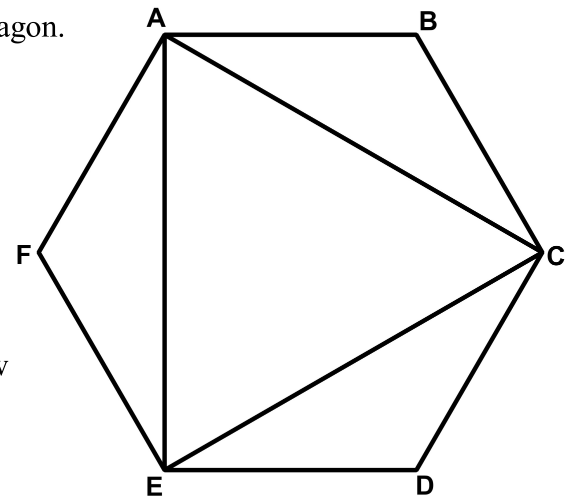
***Exercise* #2:** Let's now understand the logic behind why $ACBD$ in the last exercise must be a square. We will do this by examining four triangles created in this diagram.

(a) What triangle congruence theorem could be used to prove that $\Delta AOC \cong \Delta BOC \cong \Delta AOD \cong \Delta BOD$? Explain your choice. Because these triangles are congruent, what can you conclude about sides $\overline{AC}$, $\overline{BC}$, $\overline{BD}$ and $\overline{AD}$? Explain.

(b) Since ΔAOC (and ΔBOC and ΔBOD and ΔAOD) are all isosceles right triangles, what must be the measure of each of their acute angles? Why can you now conclude that $\angle ACB$, $\angle CBD$, $\angle BDA$ and $\angle DAC$ are 90°?

The construction of a square is relatively easy. The construction of a regular hexagon (a six-sided figure with equal side lengths and equal angles) and the construction of an equilateral triangle are tied together and are easy as well. First, let's see a relationship between the two figures.

***Exercise* #3:** In the diagram to the right, $ABCDEF$ is a regular hexagon.



(a) What triangle congruence theorem could be used to prove that:

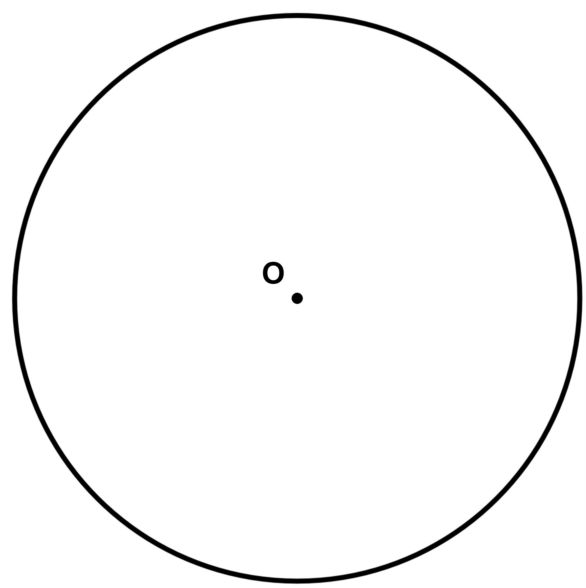
$$\Delta ABC \cong \Delta CDE \cong \Delta EFA$$

(b) Since these triangles must be congruent, why can we now conclude that ΔACE is equilateral?

We are now ready to construct both a regular hexagon and an equilateral triangle in the same construction.

***Exercise* #4:** Given the circle below with center O do the following:

(a) Draw a diameter in any direction.



(b) From each of the end points of the diameter, draw an arc whose radius is the same as the radius of the circle. Each arc should intersect the circle twice. Mark the intersection points of these arcs with the circle.

(c) Connect these intersection points and the endpoints of the diameter. Label this figure $ABCDEF$. This will be a regular hexagon.

(d) Using what we learned in #3, construct an equilateral triangle. State the name of your equilateral triangle below.

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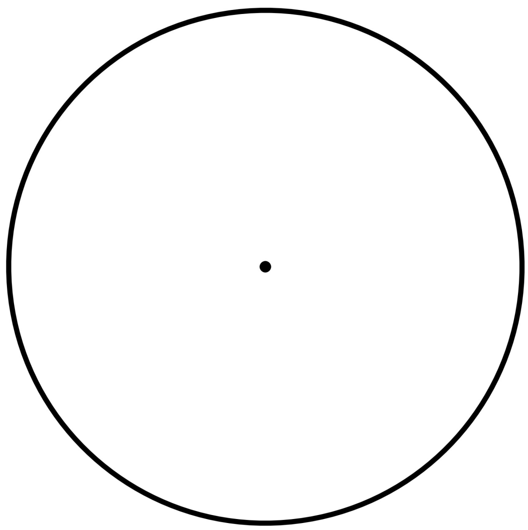
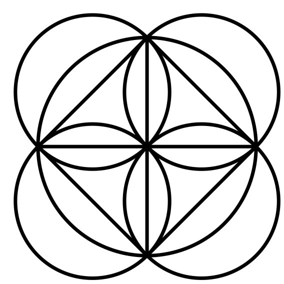
INSCRIBING REGULAR POLYGONS
COMMON CORE GEOMETRY HOMEWORK

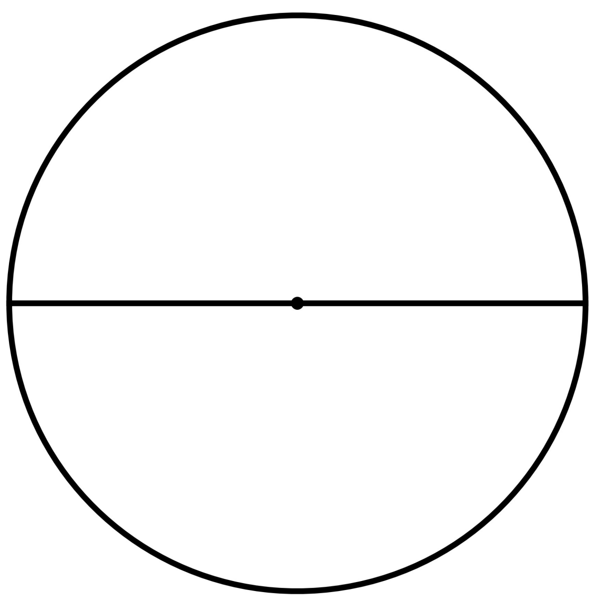
MEASUREMENT AND CONSTRUCTION

1. For the circle below, construct an inscribed square. Leave all construction marks.

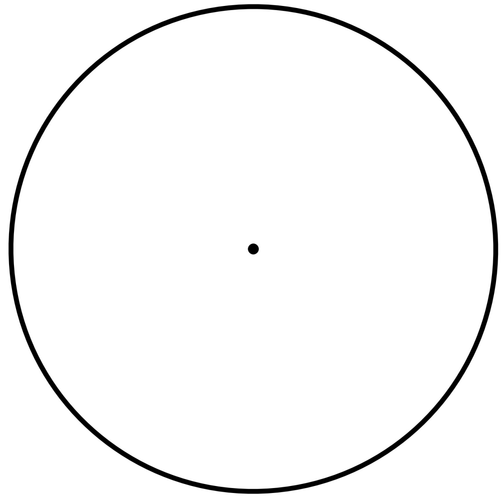


2. Using the larger circle below, construct the pattern shown. An initial horizontal diameter is drawn. You will need to construct a square and four circumscribing circles (see Lesson #4, Exercise #5). Leave all construction marks.

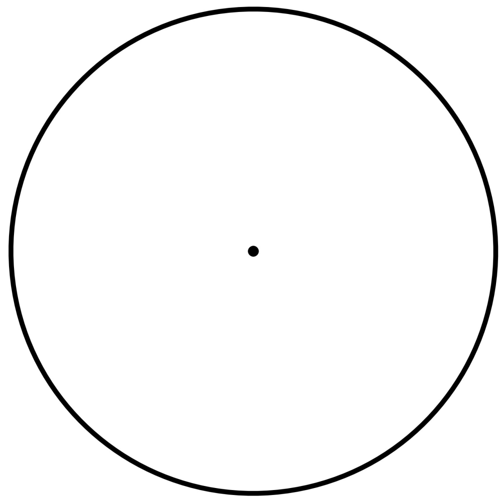




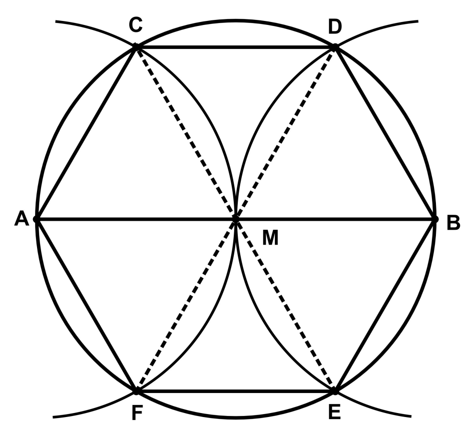
3. For the circle shown, construct a regular hexagon. Leave all construction marks.



4. For the circle shown, construct an equilateral triangle. Leave all construction marks.



5. In the diagram shown, the construction of a regular hexagon has been shown. Extra segments (normally not drawn) have been drawn in dashed.



(a) Explain why ΔACM, ΔDBM, ΔBEM, and ΔAFM are all congruent, equilateral triangles.

(b) Now, why would ΔCDM and ΔEFM also have to be equilateral and congruent to these other four?