Name: ________________________________ Date: ________________

RATIONALIZING FRACTIONS
ALGEBRA 2 WITH TRIGONOMETRY

For various historical reasons, a fraction is not considered "simplified" if it contains an irreducible square root in its denominator. The process of rationalizing, thus, is to rewrite a fraction so that it has an equivalent value, but without an irrational number in the denominator. The key to rationalizing fractions with monomial denominators lies in the first exercise.

***Exercise* #1:** Simplify each of the following products without a calculator.

(a) $\sqrt{3}\cdot\sqrt{3}$ (b) $\sqrt{7}\cdot\sqrt{7}$ (c) $\sqrt{113}\cdot\sqrt{113}$ (d) $\sqrt{a}\cdot\sqrt{a}$, for $a\geq 0$

Now, to rationalize fractions with monomial, irrational denominators, we simply multiply by the number 1 (the Multiplicative Identity Property) in a very specific form.

***Exercise* #2:** Rewrite each of the following fractions in simplest form.

(a) $\frac{2}{\sqrt{3}}$ (b) $\frac{4}{\sqrt{2}}$ (c) $\frac{5}{3\sqrt{5}}$ (d) $\sqrt{\frac{4}{7}}$

***Exercise* #3:** Which of the following is equivalent to $\sqrt{\frac{1}{2}}$?

(1) $\sqrt{2}$ (3) $\frac{1}{4}$

(2) $\frac{\sqrt{2}}{2}$ (4) $\frac{\sqrt{3}}{2}$

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Sometimes the denominator of the fraction is binomial in nature. The key to rationalizing these types of expressions comes in understanding the results of multiplying the two members of a **conjugate pair**.

***Exercise* #4:** Find each of the following products.

(a) $\left(6-\sqrt{5}\right)\left(6+\sqrt{5}\right)$ (b) $\left(2-\sqrt{7}\right)\left(2+\sqrt{7}\right)$ (c) $\left(\sqrt{10}+\sqrt{2}\right)\left(\sqrt{10}-\sqrt{2}\right)$

Because the product of the two members of a conjugate pair always follow the pattern noted below, no expression involving a square root will remain.

CONJUGATE PAIR PRODUCTS

The two binomials $(a+b)$ and $(a-b)$ are known as **conjugate pairs**. Their product is always:

$$(a+b)(a-b)=a^2-b^2$$

***Exercise* #5:** Rewrite each of the following fractions in simplest form. Be sure to both rationalize and reduce the fractions.

(a) $\dfrac{1}{3-\sqrt{7}}$

(b) $\dfrac{3}{3+\sqrt{3}}$

(c) $\dfrac{2}{4-\sqrt{6}}$

These expressions tend to become more complicated to rationalize when both the numerator and denominator are binomials. Still, simplifying these fractions is dependent on careful, but predicable, arithmetic steps.

***Exercise* #6:** Rewrite each of the following fractions in simplest form. Be sure to both rationalize and reduce the fractions.

(a) $\dfrac{3-\sqrt{5}}{3+\sqrt{5}}$

(b) $\dfrac{3+\sqrt{6}}{6-\sqrt{6}}$

***Exercise* #7:** Which of the following is equivalent to $\dfrac{10-\sqrt{2}}{4+\sqrt{2}}$?

(1) $3-\sqrt{2}$

(3) $\dfrac{19-7\sqrt{2}}{7}$

(2) $3+\sqrt{2}$

(4) $\dfrac{5-2\sqrt{2}}{2}$

Name: ___________________________________ Date: _________________

RATIONALIZING FRACTIONS
ALGEBRA 2 WITH TRIGONOMETRY - HOMEWORK

SKILLS

1. Find each of the following products. Use facts about conjugate pairs to make the multiplication faster.

(a) $\left(8-\sqrt{2}\right)\left(8+\sqrt{2}\right)$ (b) $\left(\sqrt{15}+\sqrt{5}\right)\left(\sqrt{15}-\sqrt{5}\right)$ (c) $\left(2\sqrt{2}+1\right)\left(2\sqrt{2}-1\right)$

2. Write each of the following fractions in simplest form. Be sure to both rationalize and reduce the fraction.

(a) $\dfrac{1}{\sqrt{5}}$ (b) $\dfrac{2}{\sqrt{6}}$ (c) $\dfrac{15}{\sqrt{5}}$ (d) $\dfrac{3}{2\sqrt{3}}$

(e) $\dfrac{12}{\sqrt{10}}$ (f) $\dfrac{4}{\sqrt{32}}$ (g) $\sqrt{\dfrac{1}{3}}$ (h) $\sqrt{\dfrac{2}{5}}$

3. Write each of the following fractions in simplest form. Be sure to both rationalize and reduce the fraction.

(a) $\dfrac{2}{2-\sqrt{2}}$ (b) $\dfrac{4}{\sqrt{10}-\sqrt{2}}$ (c) $\dfrac{6}{3+\sqrt{5}}$

4. Rewrite each of the following fractions in simplest form. Be sure to both rationalize and reduce the fractions.

(a) $\dfrac{\sqrt{2}+1}{\sqrt{2}-1}$

(b) $\dfrac{9+\sqrt{3}}{6-\sqrt{3}}$

5. Which of the following is equivalent to $\sqrt{\dfrac{16}{3}}$?

(1) $\dfrac{4}{3}$

(3) $\dfrac{4\sqrt{3}}{3}$

(2) $4\sqrt{3}$

(4) $\dfrac{2\sqrt{3}}{3}$

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6. Rewritten without an irrational denominator, the fraction $\dfrac{6}{5-\sqrt{7}}$ is equal to

(1) $\dfrac{5+\sqrt{7}}{3}$

(3) $\dfrac{5-\sqrt{7}}{18}$

(2) $2\sqrt{7}+5$

(4) $\dfrac{6\sqrt{7}-5}{3}$

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7. The expression $\dfrac{8-\sqrt{6}}{4-\sqrt{6}}$ can be expressed as

(1) 1

(3) $\dfrac{2+3\sqrt{6}}{5}$

(2) $\dfrac{19+2\sqrt{6}}{5}$

(4) $\dfrac{13+2\sqrt{6}}{5}$

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