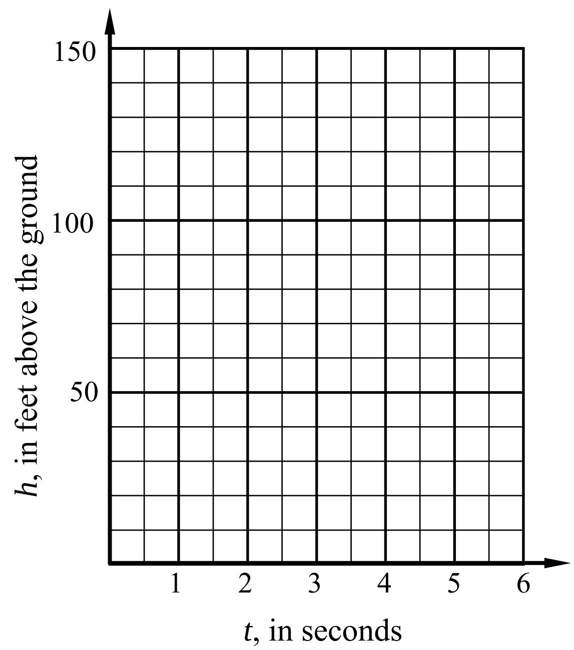
Name: ____________________________________ Date: __________________

QUADRATIC MODELING
COMMON CORE ALGEBRA I

Physical scenarios that involve **quadratic functions** occur naturally in physics, economics, and a variety of other fields. Typically, the science behind these phenomena are beyond the scope of this course, so our **quadratic modeling** is **less sophisticated** than our **linear** or **exponential.** We will take a final look, with our modeling, at some scenarios that lend themselves well to these functions.

***Exercise* #1:** Projectiles that are fired vertically into the air have heights that are quadratic functions of time. A projectile is fired from the top of a roof. It's height, in feet above the ground, after *t*-seconds is given by the function:

$$h(t) = -16(t-2)^2 + 144$$



(a) Evaluate $h(0)$. Using proper units, explain the physical significance of this answer.

(b) Determine algebraically the time when the ball hits the ground.

(c) Create a graph of $h(t)$ on the grid provided.

(d) What is the maximum height that the projectile reaches and at what time does it reach this height? Do you see this answer in the **vertex form** of the parabola?

***Exercise* #2:** Popcorn has an optimal temperature at which it pops. Food engineers at Perpetual Popping study the percent of popcorn kernels that pop at a certain temperature. Their data is shown in the table below.

Temp, *t*	385	410	440	490	510	530
Percent, *P*	38	68	78	65	45	18

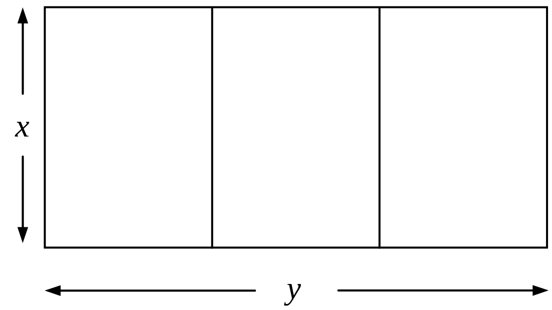
(a) Why does a quadratic model seem reasonable given the data in the table?

(b) If the engineers model the percent popped, *P*, by the equation $P = -\frac{1}{100}(t-450)^2 + 82$, then at what temperature is the greatest percent of popcorn popped? What is the greatest percent?

You can create some quadratic models just on your own from simple geometric ideas like **perimeter** and **area**. Let's do one last modeling problem that involves these two simple concepts.

***Exercise* #3:** Shana is creating a garden that has three equal sized rectangles separated by wire fencing. She has 160 feet of fencing and wants to construct the garden as shown below. Shana decides to designate the overall width of the rectangle as x and the overall length as y, as shown on the diagram.

(a) How much area would the garden contain, in square feet, if the width, x, was 10 feet? Show the calculations that lead to your answer.



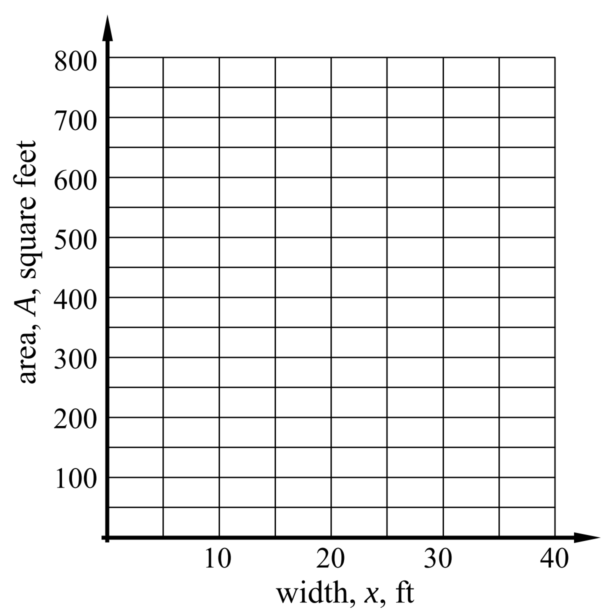
(b) Write a formula for the overall area, A, of the garden in terms of x and y. This should be a very simple formula

(c) Write an equation for the relationship between the width, x, and the length, y, based on the fact that there is 160 feet of fencing. Solve this equation for y.

(d) Find an equation for the area, A, only in terms of the width, x.

(e) Using your calculator, sketch a graph of the area function you found in (d).

(f) What is the maximum area that Shana can enclose with the 160 feet of fencing? What dimensions should she use?



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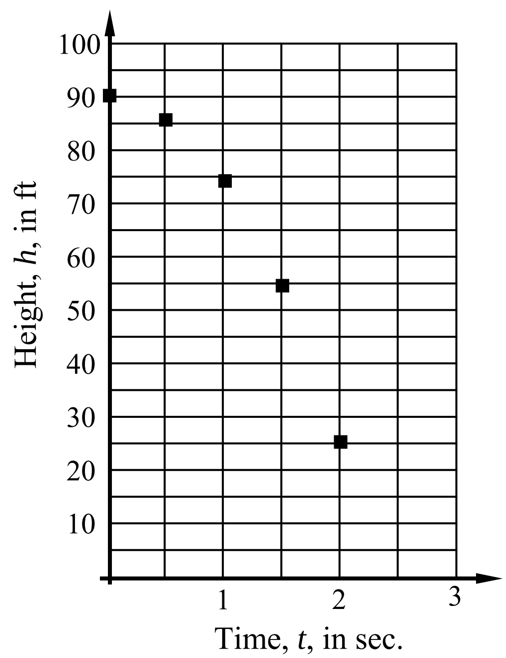
QUADRATIC MODELING
COMMON CORE ALGEBRA I HOMEWORK

APPLICATIONS

1. Physics students are modeling the height of an object dropped from the top of a 90 foot tall building. It is let go at $t=0$ seconds and using photography the students are able to measure, accurate to the nearest tenth of a foot, the height that the object is above the ground every half-second. The data is shown below.

t (sec)	0	0.5	1.0	1.5	2.0
h (ft)	90	86.1	74.2	54.5	26.8



(a) Given the scatterplot shown to the right, draw a quadratic of best fit by hand through the data. Extend your quadratic until it hits the x-axis.

(b) Students in the class approximate the equation of the quadratic of best fit by:

$h=-16t^2+90$

Calculate the residual from this model at $t=2$ seconds. Show the work that leads to your answer.

(c) Use the students' model above to determine algebraically the time, t, when the ball hits the ground. Show your work and round to the nearest tenth. How does this answer compare with where you drew the zero on the graph?

2. The Fahrenheit temperature of a chemical reaction decreases over time, measured in minutes, and then increases according to the function:

$$F(t)=\frac{1}{2}(t-8)^2+72$$

(a) For the function above, $F(0)=104$. Interpret what this means in terms of the chemical reaction.

(b) What is the minimum temperature reached during the reaction and at what time does it reach it?

3. The price of a stock rose and then fell in the span of 10 days of trading. Its price at various points in time since it was first offered is given in the table below.

Day, d	0	2	4	6	8	10
Price, p	\$30.50	\$36.50	\$38.75	\$36.75	\$30.75	\$20.50

(a) Explain why a quadratic function will model this data better than a linear or exponential function.

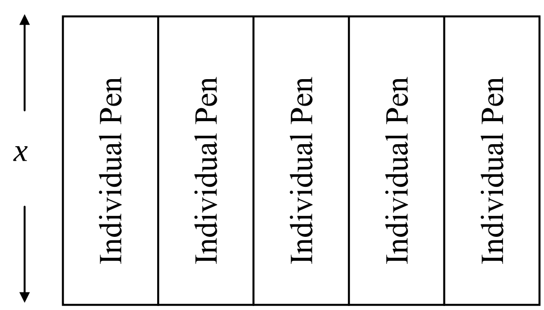
(b) If a quadratic function of the form $p = a(d-h)^2 + k$ is used to model the price, p, of the stock as a function of the day, d, then give values for h and k. Justify your choices.

(c) Which of the following value of a would be the best choice for the model given your answers to (b)? Justify your choice.

$a = -2$ $\quad$ $a = -\frac{1}{2}$ $\quad$ $a = \frac{1}{2}$ $\quad$ $a = 2$

4. A farm is creating fenced in pens and wants to lay out the pens in the following rectangular configuration where the width of the pens is given by the variable x as shown. Engineers have only 90 feet of fencing to surround and divide the pens and have created the following equation for the total area enclosed, in square feet, based on the width of x:

$A = 45x - 3x^2$



(a) Determine the zeroes of this quadratic by factoring.

(b) How can you use your answers to (a) to help determine the x-value where the maximum area will occur?

(c) Find the maximum area of the pen. Show the calculation that gives your answer.