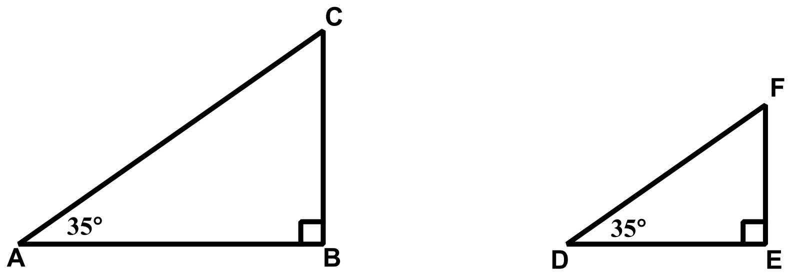
Name: ___________________________________ Date: _________________

RIGHT TRIANGLES AND SIMILARITY
COMMON CORE GEOMETRY

In our next unit, we will extensively study similar right triangles as they will be the basis for what is known as **trigonometry**. But, in this lesson we will look at an interesting example of how similar right triangls can be formed. First, though, we need to understand an important fact about right triangle similarity.

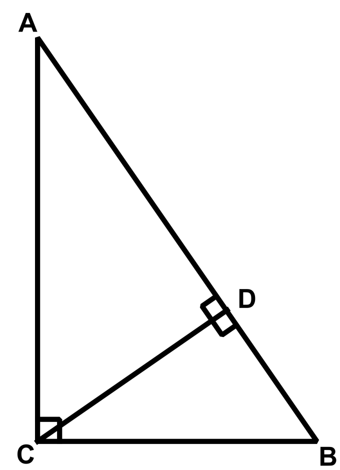
***Exercise* #1:** Given the two right triangles shown below, why do we know that ΔABC is similar to ΔDEF ?



RIGHT TRIANGLE SIMILARITY

Any two **right triangles** with a **pair of congruent acute angles** will be **similar**.

***Exercise* #2:** In the following diagram of right triangle ΔABC, the altitude has been drawn from vertex C to the hypotenuse $\overline{AB}$, creating two additional right triangles, ΔACD and ΔCBD.



(a) Why can you argue that $\Delta ABC \sim \Delta ACD$? Likewise $\Delta ABC \sim \Delta CBD$?

(b) Based on (a), why can you argue that $\Delta ACD \sim \Delta CBD$?

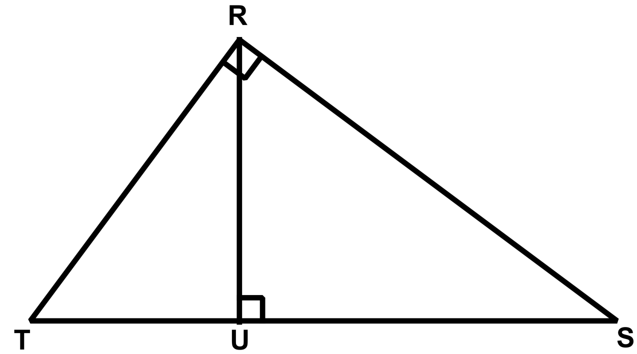
(c) Draw the two smaller triangles in the same orientation as ΔABC.

(d) If $AD = 8$ and $BD = 2$ then find the length of $\overline{CD}$ and $\overline{BC}$.
Express all answers in simplest form.

These problems can be particularly tricky, especially because there are three similar right triangles and two of the three lie inside the larger one.

***Exercise* #3:** In the diagram shown, altitude $\overline{RU}$ has been drawn to hypotenuse $\overline{ST}$ from right angle R.

(a) Draw the two smaller triangles in the same orientation as ΔRST.



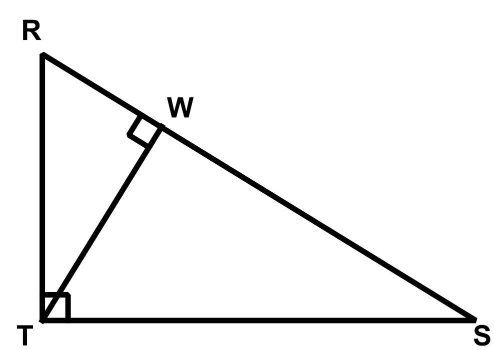
(b) If $RU = 10$ and $TU = 5$, then find the length of $\overline{US}$.

Many times, these problems are phrased without a picture. A firm control over the language and the similar triangles is critical to setting up the ratio correctly.

***Exercise* #4:** In ΔMNP, where P is a right angle, the altitude $\overline{PL}$ is drawn to hypotenuse $\overline{MN}$. If $ML = 9$ and $LN = 3$, then find the length of $\overline{PN}$.

Never forget that two figures are **similar** only if there is a **similarity transformation** that **maps** one figure onto the other. Let's try to find one for these types of right triangle scenarios.

***Exercise* #5:** In the diagram shown, altitude $\overline{TW}$ has been drawn to hypotenuse $\overline{RS}$ as shown. By the Angle-Angle Similarity Theorem ΔRST is similar to ΔRTW. Describe a similarity transformation that would map ΔRTW onto ΔRST. Be specific. Use tracing paper to help visualize the rigid motion components.



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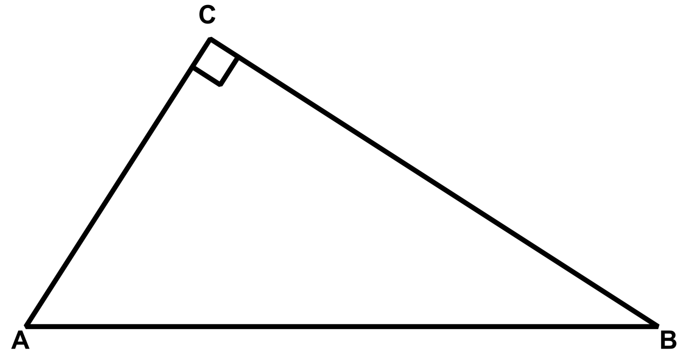
Right Triangles and Similarity
Common Core Geometry Homework

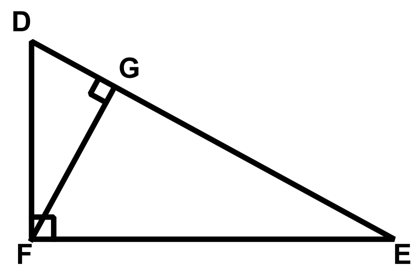
Measurement and Construction

1. Given right triangle *ABC* below with right angle *C*, construct the altitude $\overline{CD}$ to hypotenuse $\overline{AB}$. Leave all construction marks. If you can't recall how to do this construction, see Unit #4.Lesson #3.



Problem Solving

2. In the diagram of ΔDEF, the altitude from right angle $\angle DFE$ has been drawn to $\overline{DE}$. If $DG = 4$ and $GE = 8$, then which of the following is the length of $\overline{FG}$?



(1) $4\sqrt{2}$ (3) 12

(2) $3\sqrt{10}$ (4) 6

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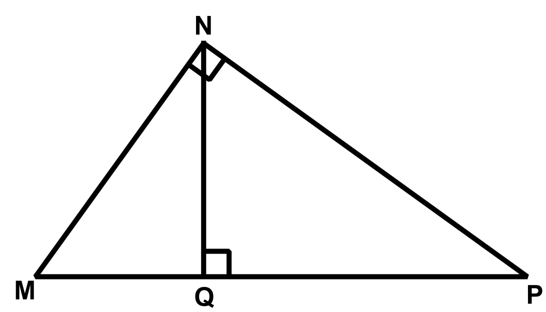
3. In the diagram below, altitude $\overline{NQ}$ has been drawn to hypotenuse $\overline{MP}$ in right triangle *MNP*. Which of the following would equal the ratio of MN to NQ?



(1) the ratio of NQ to NP
(2) the ratio of NP to MN
(3) the ratio of NP to PM
(4) the ratio of NP to PQ

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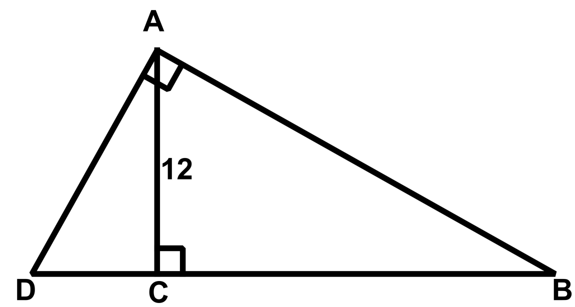
4. In right triangle QRS, $\angle R$ is a right angle. The altitude $\overline{RT}$ is drawn to hypotenuse $\overline{QS}$. If $QR = 20$ and $QS = 25$ then find the length of $\overline{QT}$. Show how you arrived at your answer.

5. If the altitude drawn to the hypotenuse of a right triangle EFG partitions it into two segments of lengths 8 and 10, then find the length of the longer leg of ΔEFG in simplest radical form.

6. In the diagram shown, altitude $\overline{AC}$ is drawn from a right angle to $\overline{BD}$ in such a way that $BC : DC = 4 : 1$. Find the length of $\overline{DC}$ algebraically. Show your work below.



Reasoning

7. In the diagram of right triangle ABC, the altitude to the hypotenuse, $\overline{CD}$, has been drawn.

(a) Justify why ΔBCD must be similar to ΔCAD.

(b) Give a similarity transformation that would map ΔBCD onto ΔCAD. Be as specific as possible. Use tracing paper to help.