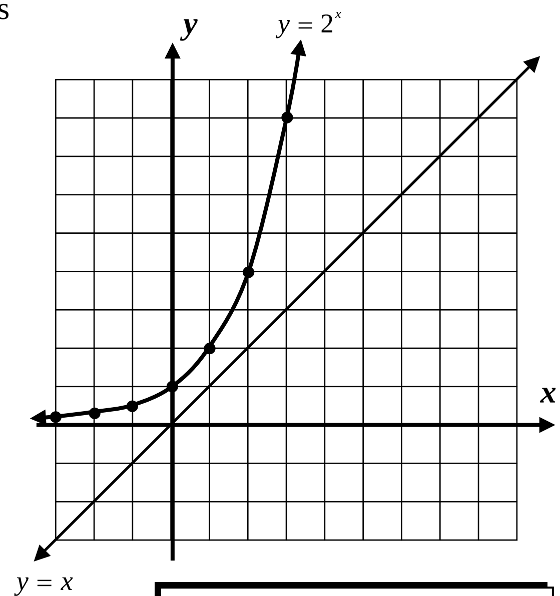
Name: ________________________________ Date: ________________

INTRODUCTION TO LOGARITHMS
ALGEBRA 2 WITH TRIGONOMETRY

Exponential functions are of such importantance to mathematics that their inverses, functions that "reverse" their action, are important themselves. These functions, known as **logarithms**, will be introduced in this lesson.

***Exercise* #1:** The function $f(x)=2^x$ is shown graphed on the axes below along with its table of values.



x	-3	-2	-1	0	1	2	3
$f(x)=2^x$	$\frac{1}{8}$	$\frac{1}{4}$	$\frac{1}{2}$	1	2	4	8

(a) Is this function one-to-one? Explain your answer.

(b) Based on your answer from part (a), what must be true about the inverse of this function?

(c) Create a table of values below for the inverse of $f(x)=2^x$ and plot this graph on the axes given.

Notice that, as always, the graphs of $f(x)$ and $f^{-1}(x)$ are symmetric across $y=x$

x							
$f^{-1}(x)$							

(d) What would be the first step to find an equation for this inverse algebraically? Write this step down and then stop.

<u>Defining Logarithmic Functions</u> – The function $y=\log_b x$ is the name we give the inverse of $y=b^x$. For example, $y=\log_2 x$ is the inverse of $y=2^x$. Based on *Exercise* #1(d), we can write an **equivalent exponential equation** for each logarithm as follows:

$$y=\log_b x \text{ is the same as } b^y=x$$

Based on this, we see that a logarithm gives as its output (y-value) the exponent we must raise b to in order to produce its input (x-value).

***Exercise* #2:** Evaluate the following logarithms. If needed, write an equivalent exponential equation. Do as many as possible without the use of your calculator.

(a) $\log_2 8$ (b) $\log_4 16$ (c) $\log_5 625$ (d) $\log_{10} 100,000$

(e) $\log_6\left(\frac{1}{36}\right)$ (f) $\log_2\left(\frac{1}{16}\right)$ (g) $\log_5 \sqrt{5}$ (h) $\log_3 \sqrt[5]{9}$

It is critically important to understand that logarithms **give exponents as their outputs**. We will be working for multiple lessons on logarithms and a basic understanding of their inputs and outputs is critical.

***Exercise* #3:** If the function $y = \log_2(x+8)+9$ was graphed in the coordinate plane, which of the following would represent its *y*-intercept?

(1) 12 (3) 8

(2) 13 (4) 9 ________

***Exercise* #4:** Between which two consecutive integers must $\log_3 40$ lie?

(1) 1 and 2 (3) 3 and 4

(2) 2 and 3 (4) 4 and 5 ________

<u>Calculator Use and Logarithms</u> – Most calculators only have two logarithms that they can evaluate directly. One of them, $\log_{10} x$, is so common that it is actually called the **common log** and typically is written without the base 10.

$$\log x = \log_{10} x \quad \text{(The Common Log)}$$

***Exercise* #5:** Evaluate each of the following using your calculator.

(a) $\log 100$ (b) $\log\left(\frac{1}{1000}\right)$ (c) $\log\sqrt{10}$

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INTRODUCTION TO LOGARITHMS
ALGEBRA 2 WITH TRIGONOMETRY - HOMEWORK

SKILLS

1. Which of the following is equivalent to $y = \log_7 x$?

(1) $y = x^7$ (3) $x = 7^y$

(2) $x = y^7$ (4) $y = x^{1/7}$

____

2. If the graph of $y = 6^x$ is reflected across the line $y = x$ then the resulting curve has an equation of

(1) $y = -6^x$ (3) $x = \log_6 y$

(2) $y = \log_6 x$ (4) $x = y^6$

____

3. The value of $\log_5 167$ is closest to which of the following? Hint – guess and check the answers.

(1) 2.67 (3) 4.58

(2) 1.98 (4) 3.18

____

4. Which of the following represents the *y*-intercept of the function $y = \log(x + 1000) - 8$?

(1) –8 (3) 3

(2) –5 (4) 5

____

5. Determine the value for each of the following logarithms. (Easy)

(a) $\log_2 32$ (b) $\log_7 49$ (c) $\log_3 6561$ (d) $\log_4 1024$

6. Determine the value for each of the following logarithms. (Medium)

(a) $\log_2\left(\frac{1}{64}\right)$ (b) $\log_3(1)$ (c) $\log_5\left(\frac{1}{25}\right)$ (d) $\log_7\left(\frac{1}{343}\right)$

7. Determine the value for each of the following logarithms. Each of these will have non-integer, fractional answers. (Difficult)

(a) $\log_4 2$ (b) $\log_4 8$ (c) $\log_5 \sqrt[3]{5}$ (d) $\log_2 \sqrt[5]{4}$

8. Between what two consecutive integers must the value of $\log_4 7342$ lie? Justify your answer.

9. Between what two consecutive integers must the value of $\log_5 \left(\frac{1}{500} \right)$ lie? Justify your answer.

APPLICATIONS

10. In chemistry, the pH of a solution is defined by the equation $\text{pH} = -\log(H)$ where H represents the concentration of hydrogen ions in the solution. Any solution with a pH less than 7 is considered acidic and any solution with a pH greater than 7 is considered basic. Fill in the table below. Round your pH's to the nearest *tenth* of a unit.

Substance	**Concentration of Hydrogen**	**pH**	**Basic or Acidic?**
Milk	1.6×10^{-7}		
Coffee	1.3×10^{-5}		
Bleach	2.5×10^{-13}		
Lemmon Juice	7.9×10^{-2}		
Rain	1.6×10^{-6}		

REASONING

11. Can the value of $\log_2(-4)$ be found? What about the value of $\log_2 0$? Why or why not? What does this tell you about the domain of $\log_b x$?