COMPOUND INEQUALITIES (THE UGLY TRUTH ABOUT AND AND OR) COMMON CORE ALGEBRA I



Linear inequalities tend to have an **infinite** amount of values for the replacement variable (typically x) that solve the inequality. Sometimes, we put two (or more) inequalities together and ask what x values make both true (AND) and which make either one or the other true (OR). You will deal with AND and OR along with **truth** values for the remainder of Algebra, so let's discuss them in an exercise.

Exercise #1: Consider each of the following compound (meaning more than one) inequality statements. Determine the truth value of both inequalities and then determine the overall truth value (or at least what you think it is).

(a) 7 > 3 and 2 < 10 (b) 5 < 10 and 11 > 20

(c) -4 < 7 or 8 < 2 (d) 3 > 6 or 8 < 5

Most students would correctly judge the truth values of the four examples above correctly. But, there is a strange disconnect in math between our use of the word **or** and the way it is used in the "real world." The next exercise will clarify this.

Exercise #2: Consider the compound inequality: 8 > 2 or 3 < 10.

- (a) Determine the truth value of each of the inequalities in this compound inequality.
- (b) What does your intuition tell you the truth value of the compound statement is? What is the mathematical truth value? This is because in mathematics we use the **inclusive or**.

English Intuition Truth Value:

Mathematical Truth Value:

Exercise #3: Which of the following compound inequalities is false? Explain your reasoning by showing the truth values of each of the individual inequalities.

(1) 5 > 2 or 4 < 1 (3) 10 > 0 or -3 < 9

(2) -6 < 5 and $7 \ge 7$ (4) $-2 \le 4$ and 5 > 7





TRUTH VALUES FOR AND AND OR

1. A compound AND statement will be **true** only if **all of the individual statements are true**.

2. A compound OR statement will be true if at least one of its individual statements is true.

Now we can start to judge the truth values of inequalities that involve algebraic expressions and replacement values. Don't ever forget that:

SOLUTIONS SETS OF EQUATIONS AND INEQUALITIES

A value of a variable is **in the solution set** of an **equation** or **inequality** if it makes it **true** and is **not** in the solution set if it makes the value **false**.

Exercise #4: Determine if each of the following values of *x* is in the solution set to the compound inequalities given below?

- (a) Is x=2 part of the solution set of x > -3 and x < 5? Justify your answer.
- (b) Is x = -4 part of the solution set of $x \le -4$ or x > 0? Justify your answer.

(d) Determine if x = 5 part of the solution set of:

2x-1 < 3 or $\frac{x+7}{2} = 6$

(c) Determine if x=1 part of the solution set of:

$$3x + 8 > 9$$
 and $-2x + 10 < 7$

Justify.

Justify.

We would also like to be able to produce number line graphs of compound inequalities. For now, we will stick with a few simple ones.

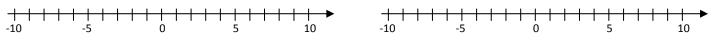
Exercise #5: On the number lines below, shade in all values of x that solve the compound inequality. In other words, shade in the compound inequalities **solution set**. If you need a good place to start, try listing some x values that make the compound inequalities true.

(a) x < 7 and $x \ge -2$

List some values:

(b) $x \ge 5$ or x < -1

List some values:





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COMPOUND INEQUALITIES (THE UGLY TRUTH ABOUT AND AND OR) COMMON CORE ALGEBRA I HOMEWORK

FLUENCY

- 1. Determine if each of the following statements is true or false. Justify your answer.
 - (a) Albany is the capital of New York and New York City is the capital of New York.
 - (b) Albany is the capital of New York or New York City is the capital of New York.
 - (c) Poughkeepsie is the capital of New York or New York City is the capital of New York.
- 2. Determine the truth value of each of the following compound inequalities by first determining the truth value of each of the individual inequalities.
 - (a) $5 \le 10$ and 3 < -4 (b) 2 < 7 or -20 > -18

(c)
$$-6 < -7$$
 or $-2 \le -2$ (d) $-5 > -8$ and $5 < 8$

- 3. Which of the following compound inequalities is true? Explain your reasoning by showing the truth values of each of the individual inequalities.
 - (1) 5 > 2 and 4 < 1 (3) -2 > 0 or $-6 \ge 6$
 - (2) $5 \le 5$ and $-6 \ge -5$ (4) $-2 \ge -4$ and 3 > 0





APPLICATIONS

4. When at a carnival there are height restrictions to go on each ride. Determine which rides each member of this family can go on by filling out the table below:

	The Swings: $h > 24$ and $h < 70$	The Twister: $h > 48$ or $h \le 60$	Wooden Rollercoaster: h > 42 and $h < 72$	Tea cups: $h \le 35$ or $h \ge 60$
Tracey: <i>h</i> = 47 inches				
Mark: <i>h</i> = 70 inches				
Marissa: h = 28 inches				

Which ride can every member go on?

REASONING

5. Determine if each of the following values of x is in the solution set to the compound inequalities given below? Justify each of your choices by showing your calculations.

(a)
$$x = 0$$
 for:
 $3x + 2 \le 12$ or $3(x+1) < -4(3x+1)$
(b) $x = 2$ for:
 $\frac{2(x+1)}{3} \le 6$ and $-2(3-2x) < 2$

(c)
$$x = -1$$
 for: (d) $x = 5$ for:

$$3x+7 < -11 \text{ or } 4-2x \le 18$$
 $\frac{2x-4}{2} \ge 3 \text{ and } \frac{x-3}{4} = 2$



